ANALYSIS OF MULTIPHASE SYSTEMS

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CERTIFICATE

Certified that this work titled "Analysis of Multiphase Systems" by Mr. Avanish Chandra Chaubey has been carried out under my supervision and has not been submitted elsewhere for a degree.

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CONTENTS

ABSTRACT

A multiphase power system network can be subdivided into two classes, one consisting of those elements which possess only rotational symmetries and another of elements which possess both rotational as well as reflection symmetries. The rotational symmetries of an n-phase network consisting of n-fold proper rotations together constitute the cyclic group C_n. The balanced n-phase stationary networks possess reflection symmetries in addition to rotational ones. These symmetries constitute a group C_{nv} consisting of n-fold rotations and n-fold reflections about its axes of symmetries. The group theoretic techniques of dealing with these symmetries have already been used to develop general transformation matrices for 3, 4 and 6-phase systems. This thesis is concerned with developing transformation matrices with complex elements, similar to symmetrical component transformation and with real elements, similar to Clarke's component transformation, for 8 and 12-phase system for the purpose of simplifying the analysis of 8 and 12-phase power system networks. In addition, expressions for sequence impedances and complex power for both 8-phase and 12-phase system, which are useful for the purpose of planning studies and fault analysis are also presented.

CHAPTER 1

INTRODUCTION

Multiphase power systems are currently of major interest on account of the fact that while the demands for electrical energy are steadily on the increase, to find new corridors on land for power system expansion is getting more and more difficult. It is essential, therefore, to try as far as possible, to meet the growing demands by appropriately augmenting the systems on the corridors already in existence. It is in this context multiphase systems have attracted recent attention [2,3]. To design adequate protection system for a power system we have to perform short circuit studies and for this we need to develop suitable methods and transformations for the analysis of such system. Suitable transformations for 4-phase and 6-phase system have been developed recently [1,4,5]. Other higher phase system which will be feasible in future are 8-phase and 12-phase. Our concern here is to extend the group theoretic techniques of analysis to 8-phase and 12-phase systems. Here we have developed the transformations for the purpose of steady state analysis of 8-phase and 12-phase systems. However, following the same procedure transformations for transient analysis can be derived.

The symmetries inherent in multiphase power system networks are:

(i) Rotational symmetries in space which correspond to physical rotations of networks, known as proper rotations.

(ii) Rotational symmetries in space followed by reflection about axis of symmetry, known as improper rotations.

Most of the components of a power system network possess rotational symmetries. However, some of the components viz. transmission and distribution networks possess reflection symmetries in addition to rotational ones.

The significant feature of these symmetries is that they satisfy group axioms so that the networks are amenable to the group theoretic techniques. The main advantage of using group theoretic techniques is that by the application of these techniques system equations of a general power system network for the purpose of steady state and transient analysis can be put into a diagonal or at least block diagonal form so that original network can be replaced by a set of smaller disjoint networks whose analysis is straight forward [1,4,5]. Now we give chapterwise description of the thesis.

In Chapter 2, it has been demonstrated that two symmetries viz. rotational symmetry and rotational symmetry followed by reflection symmetry constitute groups and hence can be represented by permutation matrices. These permutation matrices also satisfy group axioms.

Chapter 3 deals with development of suitable transformations for 8-phase power system network analysis using group theoretic

techniques. The symmetries of 8-phase power system network with rotational elements constitute a cyclic group C_8 consisting of 8-fold rotations. The transformation matrix for such network is similar to symmetrical components. The symmetries of 8-phase power system network with stationary elements constitute a group C_{8v}, consisting of 8-fold rotations and 8-fold reflections about the axes of symmetries of the network. The transformation matrix in this case has real elements and is similar to Clarke's component transformation known for 3-phase system. Sequence impedances and expression for complex power are also given.

Chapter 4 deals with development of suitable transformations for 12-phase power system network analysis applying group theoretic techniques. In this chapter, we have presented the two transformation matrices, one with complex basis and other with real basis and expressions for complex power and sequence impedances for 12-phase power system.

CHAPTER 2

SYMMETRIES IN MULTIPHASE NETWORKS

2.1 INTRODUCTION

A power system network used for generation, transmission and distribution of electric power is inherently symmetrical and balanced. These balanced multiphase power system networks can be categorised in to two classes, one which consists of only those elements which possess rotational symmetries and other which consist of elements possessing both rotational as well as reflection symmetries. The elements possessing only rotational symmetries are normally referred as rotational elements whereas the elements possessing both rotational as well as reflection symmetries, stationary elements.

A network is said to be symmetric under a symmetry operation if the system after such an operation is physically indistinguishable from the system before the symmetry operation. With every symmetry operation we can associate a symmetry property which can be attributed to the system which is symmetric under the symmetry operation in question. The two symmetries which are inherently present in the power system networks are reflection symmetry and rotational symmetry.

The reflection symmetry, also referred to as bilateral symmetry is attributed to the symmetry operation of reflection R about the symmetry axis δ which is equivalent to rotation

through 180° in space about the axis of symmetry of the system. It is obvious that the symmetry operation of reflection if applied twice, brings back the system to the original. This implies that the symmetry operation R is its own inverse. If E be the identity symmetry operation then inverse symmetry operation R^{-1} of symmetry operation of reflection R is same as R, i.e. $R.R = E$ and $R = R^{-1}$.

The rotational symmetry is attributed to the systems which are symmetric under the symmetry operation of rotation C about the axis of rotation passing through centroid O and perpendicular to the plane of the system. The admissible rotation operations will be all those rotations which bring the system into coincidence with the original system. For example, for a n-phase power system possessing rotational symmetry, rotations through $\frac{2\pi}{m}$ degrees (for $m = 1,2,\ldots,n$) are the only admissible rotations. Such rotations are referred to as proper rotation. The general symbol for a proper axis of rotation is C_n where the subscript n denotes the order of the axis. By order is meant the largest value of n such that rotation through $\frac{2\pi}{n}$ gives an equivalent configuration. A rotation by $\frac{2\pi}{n}$ degrees about proper axis C_n is also represented by the symbol C_n. Rotation by $\frac{2\pi}{n}$ degrees carried out m times successively is represented by the symbol C_n^m. It is obvious that rotation C_n^n is rotation by $n\frac{2\pi}{n} = 2\pi$ degrees and hence equal to zero rotation or identity operation E, i.e. $C_n^n = E$. It can be further verified that $C_n^{n+1} = C_n^1 = C_n$ and $C_n^{n+2} = C_n^2$ ... etc.

Therefore, it can be observed that a proper axis of order n generates n symmetry operations viz. C_n^1, C_n^2, $\ldots, C_n^n (=E)$.

2.2 MULTIPHASE POWER SYSTEM NETWORKS WITH ROTATIONAL SYMMETRY

a) 8-Phase Symmetric Element:

Let us consider a 8-phase symmetric (i.e. balanced) power system subnetwork between buses p and q as shown in Figure 2.1. The steady state network equation in the impedance form of this subnetwork will be,

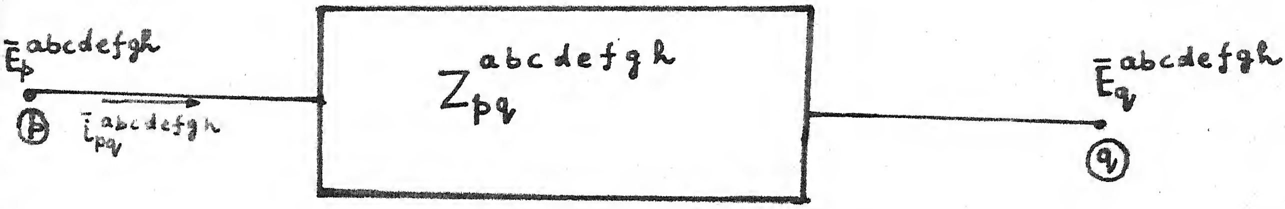
$$\bar{E}_p^{abcdefgh} - [Z_{pq}]^{abcdefgh}\, \bar{i}_{pq}^{abcdefgh} = \bar{E}_q^{abcdefgh}$$

i.e. $$\bar{E}_p^{abcdefgh} - \bar{E}_q^{abcdefgh} = [Z_{pq}]^{abcdefgh}\, \bar{i}_{pq}^{abcdefgh}$$

$$= \bar{v}_{pq}^{abcedfgh} \qquad (2.1)$$

where $\bar{V}_{pq}^{abcdefgh}$ is the column vector of voltage drops across the 8-phase element p-q, $\bar{E}_p^{abcdefgh}$ and $\bar{E}_q^{abcdefgh}$ are the column vectors of bus voltages for the buses p and q respectively and $[Z_{pq}]^{abcdefgh}$ is a 8x8 impedance matrix of a 8-phase element p-q. The equation (2.1) can be expressed as

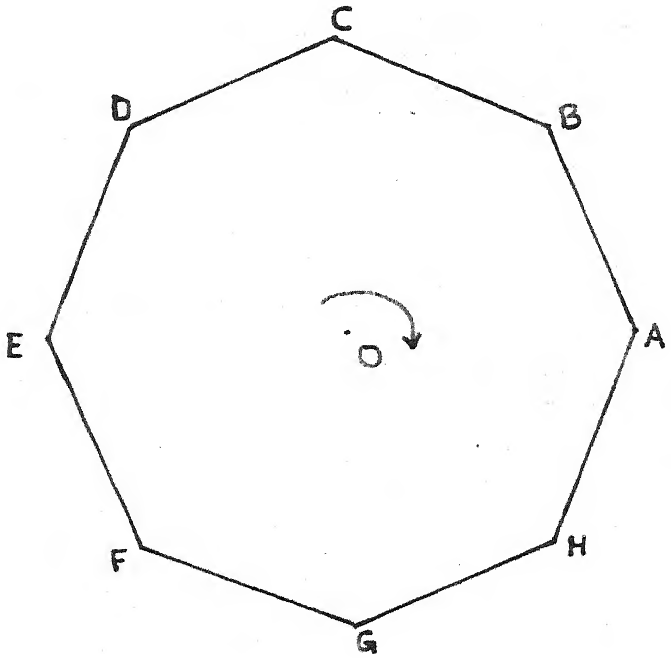
$$\begin{bmatrix} v_{pq}^a \\ v_{pq}^b \\ v_{pq}^c \\ v_{pq}^d \\ v_{pq}^e \\ v_{pq}^f \\ v_{pq}^g \\ v_{pq}^h \end{bmatrix} = \begin{bmatrix} z_{pq}^{aa} & z_{pq}^{ab} & z_{pq}^{ac} & z_{pq}^{ad} & z_{pq}^{ae} & z_{pq}^{af} & z_{pq}^{ag} & z_{pq}^{ah} \\ z_{pq}^{ba} & z_{pq}^{bb} & z_{pq}^{bc} & z_{pq}^{bd} & z_{pq}^{be} & z_{pq}^{bf} & z_{pq}^{bg} & z_{pq}^{bh} \\ z_{pq}^{ca} & z_{pq}^{cb} & z_{pq}^{cc} & z_{pq}^{cd} & z_{pq}^{ce} & z_{pq}^{cf} & z_{pq}^{cg} & z_{pq}^{ch} \\ z_{pq}^{da} & z_{pq}^{db} & z_{pq}^{dc} & z_{pq}^{dd} & z_{pq}^{de} & z_{pq}^{df} & z_{pq}^{dg} & z_{pq}^{dh} \\ z_{pq}^{ea} & z_{pq}^{eb} & z_{pq}^{ec} & z_{pq}^{ed} & z_{pq}^{ee} & z_{pq}^{ef} & z_{pq}^{eg} & z_{pq}^{eh} \\ z_{pq}^{fa} & z_{pq}^{fb} & z_{pq}^{fc} & z_{pq}^{fd} & z_{pq}^{fe} & z_{pq}^{ff} & z_{pq}^{fg} & z_{pq}^{fh} \\ z_{pq}^{ga} & z_{pq}^{gb} & z_{pq}^{gc} & z_{pq}^{gd} & z_{pq}^{ge} & z_{pq}^{gf} & z_{pq}^{gg} & z_{pq}^{gh} \\ z_{pq}^{ha} & z_{pq}^{hb} & z_{pq}^{hc} & z_{pq}^{hd} & z_{pq}^{he} & z_{pq}^{hf} & z_{pq}^{hg} & z_{pq}^{hh} \end{bmatrix} \begin{bmatrix} i_{pq}^a \\ i_{pq}^b \\ i_{pq}^c \\ i_{pq}^d \\ i_{pq}^e \\ i_{pq}^f \\ i_{pq}^g \\ i_{pq}^h \end{bmatrix} \qquad (2.2)$$



$$\bar{v}_{pq}^{abcdefgh} = \bar{E}_p^{abcdefgh} - \bar{E}_q^{abcdefgh}$$

8-Phase element p-q

FIGURE 2.1



Symmetry Group C_8

FIGURE 2.2

For rotating elements, the symmetries are such that the circularly permuting the port voltages will cause similar permutation of the port currents. These symmetries may equivalently be represented by the physical rotation of the network element as shown diagrammet. in Figure 2.2 in which vertices ABCDEFGH represent the eight phases. The element shown in this figure is regular octagon with O as centroid. The axis of rotation "δ" passes through O and is perpendicular to the plane of the paper. Considering all possible rotations of octagon about axis δ. We can see only eight rotations out of all possible rotations through different angles (less than or equal to 360°) send the edges and vertices of octagon into coincidence. Therefore, the admissible rotations are rotations through 45°, 90°, 135°, 180°, 225°, 270°, 315° and 360° (= 0) about axis δ passing through centroid O and perpendicular to the plane of the paper.

The rotation through 360° which sends the vertices back to their original position is also referred as zero rotation. The permutation table for the zero rotation is given below:

Rotation through 360°								
	A	B	C	D	E	F	G	H
	A	B	C	D	E	F	G	H

The first clockwise rotation through 45° about axis δ sends the vertex A to H, B to A, C to B, D to C, E to D, F to E, G to F and H to G. This rotation through 45° can be represented by following permutation table:

Rotation through 45°								
	A	B	C	D	E	F	G	H
	H	A	B	C	D	E	F	G

Similarly, other rotations can be represented by the permutation tables given below:

Rotation through 90°								
	A	B	C	D	E	F	G	H
	G	H	A	B	C	D	E	F

Rotation through 135°								
	A	B	C	D	E	F	G	H
	F	G	H	A	B	C	D	E

Rotation through 180°								
	A	B	C	D	E	F	G	H
	E	F	G	H	A	B	C	D

Rotation through 225°								
	A	B	C	D	E	F	G	H
	D	E	F	G	H	A	B	C

Rotation through 270°								
	A	B	C	D	E	F	G	H
	C	D	E	F	G	H	A	B

and

Rotation through 315°								
	A	B	C	D	E	F	G	H
	B	C	D	E	F	G	H	A

If we use the general symbol for proper axis of rotation and rotations, then C_8 is the proper axis of rotation and symmetry operations; C_8^1, C_8^2, C_8^3, C_8^4, C_8^5, C_8^6, C_8^7 and C_8^8 (= E) represent the rotations through 45°, 90°, 135°, 180°, 225°, 270°, 315° and 360° respectively.

We know that the product of two rotations is same as application of the two rotations successively. It can be seen that application of symmetry operation C_8^1 (rotation through 45°) twice is same as C_8^2 (rotation through 90°), i.e.

$$C_8^1 . C_8^1 = C_8^2$$

The multiplication of all these symmetry operation can be compactly expressed in the form of a group multiplication table, known as Cayley table as shown in Table 1. In this table the entry (a_{ij}) in the ith row and jth column is the product of $a_i . a_j$.

Table 1

	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7
E	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7
C_8^1	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	E
C_8^2	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	E	C_8^1
C_8^3	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	E	C_8^1	C_8^2
C_8^4	C_8^4	C_8^5	C_8^6	C_8^7	E	C_8^1	C_8^2	C_8^3
C_8^5	C_8^5	C_8^6	C_8^7	E	C_8^1	C_8^2	C_8^3	C_8^4
C_8^6	C_8^6	C_8^7	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5
C_8^7	C_8^7	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6

Looking at table 1, it is easily verified that symmetry operations E, C_8^1, C_8^2, C_8^3, C_8^4, C_8^5, C_8^6 and C_8^7 satisfy the group axioms (Appendix 1) and commutative law. The element E serves as an identity element and C_8^1 is the generator element. Hence, we conclude that the symmetry operations E, C_8^1, C_8^2, ... and C_8^7 constitute a cyclic group C_8 of order 8. As all the symmetry operations belong to a separate class, the number of classes in the group is also 8.

Now we know that finite groups can be represented by set of matrices $D(R)$ (Appendix 1) such that for every member R of G there is a matrix $D(R)$ in it and there exists a correspondence between the matrices and the group elements such that for any R_1 and R_2 in G,

$$D(R_1.R_2) = D(R_1).D(R_2).$$

The representation matrices $D(R)$ for $R = E$, C_8^1, C_8^2, C_8^3, C_8^4, C_8^5, C_8^6 and C_8^7 are as shown below:

$$D(E) = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix} \; ; \quad D(C_8^1) = \begin{bmatrix} 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \end{bmatrix}$$

$$D(C_8^2) = \begin{bmatrix} 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{bmatrix} ; \quad D(C_8^3) = \begin{bmatrix} 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \end{bmatrix}$$

$$D(C_8^4) = \begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix} ; \quad D(C_8^5) = \begin{bmatrix} 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \end{bmatrix}$$

$$D(C_8^6) = \begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \end{bmatrix} \text{ and } D(C_8^7) = \begin{bmatrix} 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \end{bmatrix} \qquad (2.3)$$

These representation matrices are called permutation matrices.

The permutation matrix A has the following properties:

1. $A^T = A^{-1}$

2. $\det A = 1$

In other words, a permutation matrix is orthogonal. Now,

$$D(C_8^1)D(C_8^1) = \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0 \end{bmatrix} \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0 \end{bmatrix}$$

$$= \begin{bmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0 \end{bmatrix} = D(C_8^2)$$

Similarly, we can show that

$$D(C_8^1)D(C_8^2) = D(C_8^1)D(C_8^1)D(C_8^1) = D(C_8^3)$$

$$D(C_8^1)D(C_8^3) = D(C_8^1)D(C_8^1)D(C_8^1)D(C_8^1) = D(C_8^4)$$

$$D(C_8^1)D(C_8^4) = D(C_8^5)$$

$$D(C_8^1)D(C_8^5) = D(C_8^6)$$

$$D(C_8^1)D(C_8^6) = D(C_8^7)$$

$$D(C_8^1)D(C_8^7) = D(C_8^8 = E)$$

It can be verified that

$$D(C_8^2)D(C_8^1) = D(C_8^1)D(C_8^2) = D(C_8^3)$$

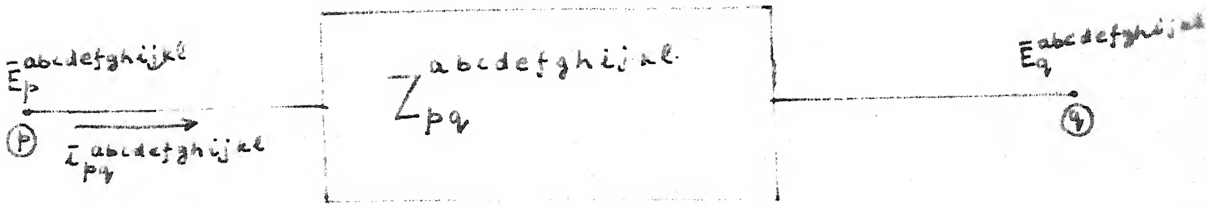
Similarly finding other multiplications of permutation matrices, we can see that the permutation matrices D(R) themselves constitute a cyclic group under multiplication having same multiplication table as for group elements of C_8. The element $D(C_8^1)$ is the generator element.

b) 12-Phase Symmetry Element:

Now let us consider a 12-phase symmetric power system subnetwork between buses p and q as shown in Figure 2.3. The steady state network equation in the impedance form of this subnetwork is,

$$\bar{E}_p^{abcdefghijkl} - \bar{E}_q^{abcdefghijkl} = [Z_{pq}]^{abcdefghijkl}\ \bar{I}_{pq}^{abcedfghijkl}$$
$$= \bar{v}_{pq}^{abcdefghijkl} \qquad (2.4)$$

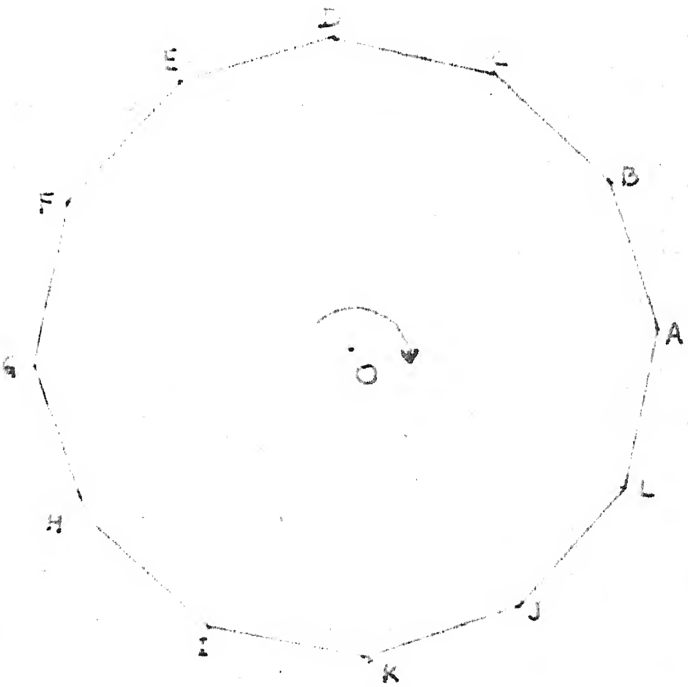
For rotating elements, the symmetries of this network may equivalently be represented by the physical rotation of the regular 12-gon as shown in Figure 2.4. The vertices ABCDEFGHIJKL signify the 12 phases of the network. The axis of rotation δ passes through the centroid O and is perpendicular



$$\bar{V}_{pq}^{abcdefghijkl} = \bar{E}_p^{abcdefghijkl} - \bar{E}_q^{abcdefghijkl}$$

12-Phase element p-q

FIGURE 2.3



Symmetry Group C_{12}

FIGURE 2.4

to the plane of the paper. After considering all possible rotations through different angles we find that the admissible rotations about axis δ are rotations through 30°, 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330° and 360°. The permutation table for these admissible (clockwise rotations) are given below.

Rotation through 360° or zero rotation

A	B	C	D	E	F	G	H	I	J	K	L
A	B	C	D	E	F	G	H	I	J	K	L

Rotation through 30°

A	B	C	D	E	F	G	H	I	J	K	L
L	A	B	C	D	E	F	G	H	I	J	K

Rotation through 60°

A	B	C	D	E	F	G	H	I	J	K	L
K	L	A	B	C	D	E	F	G	H	I	J

Rotation through 90°

A	B	C	D	E	F	G	H	I	J	K	L
J	K	L	A	B	C	D	E	F	G	H	I

Rotation through 120°

A	B	C	D	E	F	G	H	I	J	K	L
I	J	K	L	A	B	C	D	E	F	G	H

Rotation through 150°

A	B	C	D	E	F	G	H	I	J	K	L
H	I	J	K	L	A	B	C	D	E	F	G

Rotation through 180°												
	A	B	C	D	E	F	G	H	I	J	K	L
	G	H	I	J	K	L	A	B	C	D	E	F

Rottion through 210°												
	A	B	C	D	E	F	G	H	I	J	K	L
	F	G	H	I	J	K	L	A	B	C	D	E

Rotation through 240°												
	A	B	C	D	E	F	G	H	I	J	K	L
	E	F	G	H	I	J	K	L	A	B	C	D

Rotation through 270°												
	A	B	C	D	E	F	G	H	I	J	K	L
	D	E	F	G	H	I	J	K	L	A	B	C

Rotation through 300°												
	A	B	C	D	E	F	G	H	I	J	K	L
	C	D	E	F	G	H	I	J	K	L	A	B

Rotation through 330°												
	A	B	C	D	E	F	G	H	I	J	K	L
	B	C	D	E	F	G	H	I	J	K	L	A

We can see that the axis of rotation is 12-fold axis and is designated by C_{12}. The rotations through 30°, 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330° and 360° are represented by symmetry operations C_{12}^1, C_{12}^2, C_{12}^3, C_{12}^4, C_{12}^5, C_{12}^6, C_{12}^7, C_{12}^8, C_{12}^9, C_{12}^{10}, C_{12}^{11} and $C_{12}^{12} = E$ respectively. The multiplications of all these symmetry operations (rotations) can be expressed by Cayley table given in Table 2.

	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}
E	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}
C_{12}^{1}	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E
C_{12}^{2}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}
C_{12}^{3}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}
C_{12}^{4}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}
C_{12}^{5}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}
C_{12}^{6}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}
C_{12}^{7}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}
C_{12}^{8}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}
C_{12}^{9}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}
C_{12}^{10}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}
C_{12}^{11}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}

<u>Table 2</u>

Looking at table 2, it can be verified that these symmetry operations commute and satisfy the group axioms. The element E is identity element and element C_{12}^{1} is the generator element. Hence, the symmetry operations C_{12}^{1}, C_{12}^{2}, ...,C_{12}^{11},E constitute a cyclic group C_{12} of order 12. The number of classes is 12 as all the group elements in the cyclic group

The representation matrices D(R) representing group elements of group C_{12} for $R = E, C_{12}^1, C_{12}^2, \ldots, C_{12}^{11}$ are given below:

$$D(C_{12}^{12} = E) = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$D(C_{12}^1) = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix}$$

$$D(C^2_{12}) = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0
\end{bmatrix}$$

$$D(C^3_{12}) = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0
\end{bmatrix}$$

$$D(C_{12}^{4}) = \begin{bmatrix} 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0 \end{bmatrix}$$

$$D(C_{12}^{5}) \quad \begin{bmatrix} 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0 \end{bmatrix}$$

$$D(\sigma_{12}^{6}) = \begin{bmatrix} 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}$$

$$D(\sigma_{12}^{7}) = \begin{bmatrix} 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \end{bmatrix}$$

$$D(C_{12}^{8}) = \begin{bmatrix}
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0
\end{bmatrix}$$

$$D(C_{12}^{9}) = \begin{bmatrix}
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

$$D(C_{12}^{10}) = \begin{bmatrix} 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

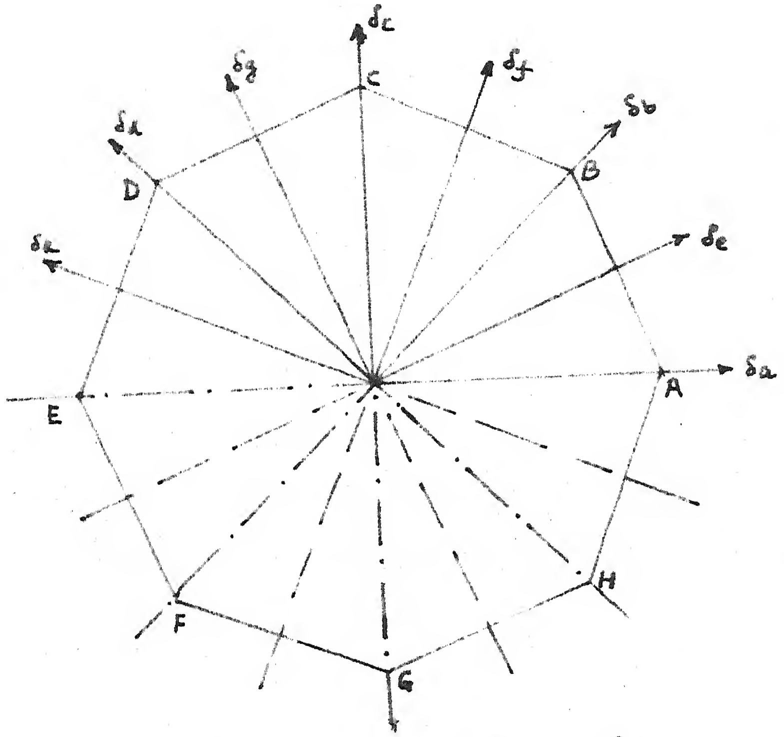
$$D(C_{12}^{11}) = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix} \qquad (2.5)$$

These representation matrices $D(R)$ themselves constitute a cyclic group of order 12 and have $D(C_{12}^{1})$ as generator element of the group.

2.3 MULTIPHASE POWER SYSTEM NETWORKS WITH ROTATIONAL AND REFLECTION SYMMETRY

a) 8-Phase network:

We now consider 8-phase symmetric power system networks which possesses in addition to rotational, reflection symmetries also. Symmetries of such network which are normally referred to as stationary element, can be equivalently represented by physical rotation of the network shown in Figure 2.5, about the axis passing through the centroid and perpendicular to the plane of the paper and also reflections of the network about the axes of symmetries. The network configuration remains invariant under rotations by 45°, 90°, 135°, 180°, 225°, 270°, 315° and 360° and also under reflection about axes of symmetry. The axes of symmetries in this case of the network in Figure 2.5, which is a regular octagon are lines joining opposite vertices andlines joining mid-points of opposite edges. These axes of symmetries are denoted by δ_a, δ_b, δ_c, δ_d, δ_e, δ_f, δ_g, and δ_h in the figure. Fornetworks with stationary elements, in all we have sixteen symmetry operations viz. C_8^1, C_8^2, C_8^3, C_8^4, C_8^5, C_8^6, C_8^7, and E representing rotations through 45°, 90°, 135°, 180°, 225°, 270°, 315° and 360° respectively and δ'_a, δ'_b, δ'_c, δ'_d, δ'_e, δ'_f, δ'_g and δ'_h representing reflections about axes of symmetries δ_a, δ_b, δ_c, δ_d, δ_e, δ_f, δ_g, and δ_h respectively. The Cayley's multiplication table for these sixteen symmetry operations is given in Table 3.



Symmetric Group C_{8v}

FIGURE 2.5

TABLE 3

	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f	δ'_g	δ'_h
E	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f	δ'_g	δ'_h
C_8^1	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	E	δ'_e	δ'_f	δ'_g	δ'_h	δ'_b	δ'_c	δ'_d	δ'_a
C_8^2	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	E	C_8^1	δ'_b	δ'_c	δ'_d	δ'_a	δ'_f	δ'_g	δ'_h	δ'_e
C_8^3	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	E	C_8^1	C_8^2	δ'_f	δ'_g	δ'_h	δ'_e	δ'_c	δ'_d	δ'_a	δ'_b
C_8^4	C_8^4	C_8^5	C_8^6	C_8^7	E	C_8^1	C_8^2	C_8^3	δ'_c	δ'_d	δ'_a	δ'_b	δ'_g	δ'_h	δ'_e	δ'_f
C_8^5	C_8^5	C_8^6	C_8^7	E	C_8^1	C_8^2	C_8^3	C_8^4	δ'_g	δ'_h	δ'_e	δ'_f	δ'_d	δ'_a	δ'_b	δ'_c
C_8^6	C_8^6	C_8^7	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	δ'_d	δ'_a	δ'_b	δ'_c	δ'_h	δ'_e	δ'_f	δ'_g
C_8^7	C_8^7	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	δ'_h	δ'_e	δ'_f	δ'_g	δ'_a	δ'_b	δ'_c	δ'_d
δ'_a	δ'_a	δ'_e	δ'_b	δ'_f	δ'_c	δ'_g	δ'_d	δ'_h	E	C_8^2	C_8^4	C_8^6	C_8^1	C_8^3	C_8^5	C_8^7
δ'_b	δ'_b	δ'_f	δ'_c	δ'_g	δ'_d	δ'_h	δ'_a	δ'_e	C_8^6	E	C_8^2	C_8^4	C_8^7	C_8^1	C_8^3	C_8^5
δ'_c	δ'_c	δ'_g	δ'_d	δ'_h	δ'_a	δ'_e	δ'_b	δ'_f	C_8^4	C_8^6	E	C_8^2	C_8^5	C_8^7	C_8^1	C_8^3
δ'_d	δ'_d	δ'_h	δ'_a	δ'_e	δ'_b	δ'_f	δ'_c	δ'_g	C_8^2	C_8^4	C_8^6	E	C_8^3	C_8^5	C_8^7	C_8^1
δ'_e	δ'_e	δ'_b	δ'_f	δ'_c	δ'_g	δ'_d	δ'_h	δ'_a	C_8^7	C_8^1	C_8^3	C_8^5	E	C_8^2	C_8^4	C_8^6
δ'_f	δ'_f	δ'_c	δ'_g	δ'_d	δ'_h	δ'_a	δ'_e	δ'_d	C_8^5	C_8^7	C_8^1	C_8^3	C_8^6	E	C_8^2	C_8^4
δ'_g	δ'_g	δ'_d	δ'_h	δ'_a	δ'_e	δ'_b	δ'_f	δ'_c	C_8^3	C_8^5	C_8^7	C_8^1	C_8^4	C_8^6	E	C_8^2
δ'_h	δ'_h	δ'_a	δ'_e	δ'_b	δ'_f	δ'_c	δ'_g	δ'_d	C_8^1	C_8^3	C_8^5	C_8^7	C_8^2	C_8^4	C_8^6	E

Looking at Table 3, we verify that these symmetry operations constitute a symmetry group C_{8v}. The group elements are eight rotation operations and eight reflection operations about symmetry axes. The identity element of the group is E.

These symmetry operations which constitute the group C_{8v}, can be represented by permutation matrices. The permutation matrices representing symmetry operation of rotations are same as given in Eqn.(2.3). The permutation matrices representing reflections about axes of symmetries viz. δ'_a, δ'_b, δ'_c, δ'_d, δ'_e, δ'_f, δ'_g and δ'_h are given below.

$$D(\delta'_a)=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&1&0&0&0&0&0&0\end{bmatrix} \qquad D(\delta'_b)=\begin{bmatrix}0&0&1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&1&0&0&0&0\end{bmatrix}$$

$$D(\delta'_c)=\begin{bmatrix}0&0&0&0&1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&1&0&0\end{bmatrix} \qquad D(\delta'_d)=\begin{bmatrix}0&0&0&0&0&0&1&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1\end{bmatrix}$$

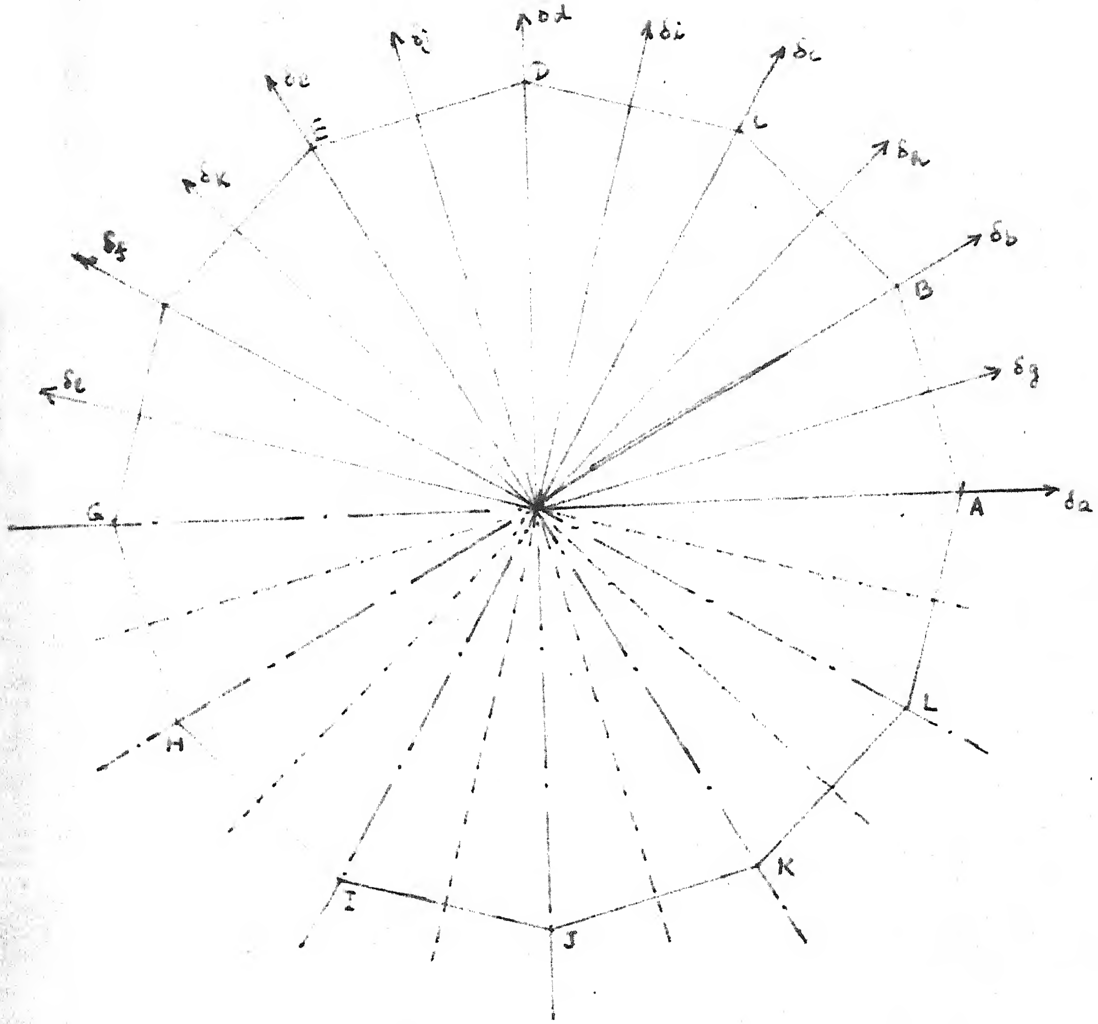
$$D(\delta'_e) = \begin{bmatrix} 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0 \end{bmatrix} \qquad D(\delta'_f) = \begin{bmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0 \end{bmatrix}$$

$$D(\delta'_g) = \begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0 \end{bmatrix} \qquad D(\delta'_h) = \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0 \end{bmatrix} \qquad (2.6)$$

It can be easily verified that these permutation matrices also constitute a group under multiplicationhaving same multiplication table as that for corresponding elements of group C_{8v}.

b) 12-Phase network:

The symmetries of 12-phase power system network **with stationary** elements can be represented by physical rotation of the network shown in Figure 2.6 about the axis passing through centroid and perpendicular to the plane of the paper and reflections of the network about the axes of symmetries.



Symmetry Group C_{12v}

FIGURE 2.6

The axis of rotation is 12-fold axis in this case and the rotations are through 30°, 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330° and 360° represented by symmetry group elements C_{12}^{1}, C_{12}^{2}, ..., C_{12}^{11} and $C_{12}^{12} = E$ respectively. The axes of reflection are lines joining opposite vertices viz. δ_a, δ_b, δ_c, δ_d, δ_e, and δ_f and lines joining mid points of the opposite edges viz. δ_g, δ_h, δ_i, δ_j, δ_k, and δ_l. The symmetry operations representing reflections about δ_a, δ_b, δ_c, δ_d, δ_e, δ_f, δ_g, δ_h, δ_i, δ_j, δ_k and δ_l are δ_a', δ_b', δ_c', δ_d', δ_e', δ_f', δ_g', δ_h', δ_i', δ_j', δ_k' and δ_l' respectively. The Cayley's multiplication table for these symmetry operations is given in Table 4. Looking at the Table 4 we can verify that these twenty four symmetry operations constitute a symmetry group C_{12v}. The group elements are twelve symmetry operations of rotation and twelve symmetry operations of reflection.

Just like 8-phase symmetric systems, these symmetry operations can be represented by permutation matrices. The permutation matrices representing rotation operations are same as given in Eqn.(2.5). The permutation matrices representing reflection operations are given in Eqn.(2.7).

It can be verified that these permutation matrices also constitute a group under multiplication having the same multiplication table as that for corresponding elements of group C_{12v}.

E	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f	δ'_g	δ'_h	δ'_i	δ'_j	δ'_k	δ'_l
C_{12}^{1}	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	δ'_g	δ'_h	δ'_i	δ'_j	δ'_k	δ'_l	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f	δ'_a
C_{12}^{2}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f	δ'_a	δ'_g	δ'_h	δ'_i	δ'_j	δ'_k	δ'_l
C_{12}^{3}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	δ'_h	δ'_i	δ'_j	δ'_k	δ'_l	δ'_g	δ'_c	δ'_d	δ'_e	δ'_f	δ'_a	δ'_b
C_{12}^{4}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	δ'_c	δ'_d	δ'_e	δ'_f	δ'_a	δ'_b	δ'_h	δ'_i	δ'_j	δ'_k	δ'_l	δ'_g
C_{12}^{5}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	δ'_i	δ'_j	δ'_k	δ'_l	δ'_g	δ'_h	δ'_d	δ'_e	δ'_f	δ'_a	δ'_b	δ'_c
C_{12}^{6}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	δ'_d	δ'_e	δ'_f	δ'_a	δ'_b	δ'_c	δ'_i	δ'_j	δ'_k	δ'_l	δ'_g	δ'_h
C_{12}^{7}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	δ'_j	δ'_k	δ'_l	δ'_g	δ'_h	δ'_i	δ'_e	δ'_f	δ'_a	δ'_b	δ'_c	δ'_d
C_{12}^{8}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	δ'_e	δ'_f	δ'_a	δ'_b	δ'_c	δ'_d	δ'_j	δ'_k	δ'_l	δ''_g	δ'_h	δ'_i
C_{12}^{9}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	δ'_k	δ'_l	δ'_g	δ'_h	δ'_i	δ'_j	δ'_f	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e
C_{12}^{10}	C_{12}^{10}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	δ'_f	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e	δ'_k	δ'_l	δ'_g	δ'_h	δ'_i	δ'_j
C_{12}^{11}	C_{12}^{11}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	δ'_l	δ'_g	δ'_h	δ'_i	δ'_j	δ'_k	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f

Continued ...

Table 4 continued

δ'_a	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	E	C_{12}^2	C_{12}^4	C_{12}^6	C_{12}^8	C_{12}^{10}	C_{12}^1	C_{12}^3	C_{12}^5	C_{12}^7	C_{12}^9	C_{12}^{11}
δ'_b	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	C_{12}^{10}	E	C_{12}^2	C_{12}^4	C_{12}^6	C_{12}^8	C_{12}^{11}	C_{12}^1	C_{12}^3	C_{12}^5	C_{12}^7	C_{12}^9
δ'_c	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	C_{12}^8	C_{12}^{10}	E	C_{12}^2	C_{12}^4	C_{12}^6	C_{12}^9	C_{12}^{11}	C_{12}^1	C_{12}^3	C_{12}^5	C_{12}^7
δ'_d	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	C_{12}^6	C_{12}^8	C_{12}^{10}	E	C_{12}^2	C_{12}^4	C_{12}^7	C_{12}^9	C_{12}^{11}	C_{12}^1	C_{12}^3	C_{12}^5
δ'_e	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	C_{12}^4	C_{12}^6	C_{12}^8	C_{12}^{10}	E	C_{12}^2	C_{12}^5	C_{12}^7	C_{12}^9	C_{12}^{11}	C_{12}^1	C_{12}^3
δ'_f	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	C_{12}^2	C_{12}^4	C_{12}^6	C_{12}^8	C_{12}^{10}	E	C_{12}^3	C_{12}^5	C_{12}^7	C_{12}^9	C_{12}^{11}	C_{12}^1
δ'_g	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	C_{12}^{11}	C_{12}^1	C_{12}^3	C_{12}^5	C_{12}^7	C_{12}^9	E	C_{12}^2	C_{12}^4	C_{12}^6	C_{12}^8	C_{12}^{10}
δ'_h	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	C_{12}^9	C_{12}^{11}	C_{12}^1	C_{12}^3	C_{12}^5	C_{12}^7	C_{12}^{10}	E	C_{12}^2	C_{12}^4	C_{12}^6	C_{12}^8
δ'_i	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	C_{12}^7	C_{12}^9	C_{12}^{11}	C_{12}^1	C_{12}^3	C_{12}^5	C_{12}^8	C_{12}^{10}	E	C_{12}^2	C_{12}^4	C_{12}^6
δ'_j	δ'_j	δ'_e	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	C_{12}^5	C_{12}^7	C_{12}^9	C_{12}^{11}	C_{12}^1	C_{12}^3	C_{12}^6	C_{12}^8	C_{12}^{10}	E	C_{12}^2	C_{12}^4
δ'_k	δ'_k	δ'_f	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	C_{12}^3	C_{12}^5	C_{12}^7	C_{12}^9	C_{12}^{11}	C_{12}^1	C_{12}^4	C_{12}^6	C_{12}^8	C_{12}^{10}	E	C_{12}^2
δ'_l	δ'_l	δ'_a	δ'_g	δ'_b	δ'_h	δ'_c	δ'_i	δ'_d	δ'_j	δ'_e	δ'_k	δ'_f	C_{12}^1	C_{12}^3	C_{12}^5	C_{12}^7	C_{12}^9	C_{12}^{11}	C_{12}^2	C_{12}^4	C_{12}^6	C_{12}^8	C_{12}^{10}	E

$$D(\varepsilon'_a) = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$D(\delta'_b) = \begin{bmatrix} 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$D(\delta_c') = \begin{bmatrix}
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0
\end{bmatrix}$$

$$D(\delta_d') = \begin{bmatrix}
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0
\end{bmatrix}$$

$$D(\delta'_e) = \begin{bmatrix} 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \end{bmatrix}$$

$$D(\delta'_f) = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$D(\delta'_g) = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$D(\delta'_h) = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0 \end{bmatrix}$$

$$D(\delta_i') = \begin{bmatrix} 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \end{bmatrix}$$

$$D(\delta_j') = \begin{bmatrix} 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \end{bmatrix}$$

$$
D(\delta'_k) = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0
\end{bmatrix}
$$

and

$$
D(\delta'_1) = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix} \tag{2.7}
$$

CHAPTER 3

EIGHT PHASE POWER SYSTEM NETWORKS

In the previous chapter we have seen that symmetries of multiphase systems can be represented by permutation matrices D(R). In this chapter, we extend the group theoretic techniques to develop suitable transformations for the purposes of steady state as well as transient analysis of 8-phase power systems.

3.1 8-PHASE ROTATING ELEMENTS

For rotating elements, the symmetries are such that circularly permuting port voltages will cause similar permutations of the port currents, i.e. if the voltage vector $\bar{v}_{pq}^{abcdefgh}$ is changed to $D(R)\bar{v}_{pq}^{abcdefgh}$ then correspondingly the current vector $\bar{i}_{pq}^{abcdefgh}$ is replaced by $D(R)\bar{i}_{pq}^{abcdefgh}$. Hence, from equation (2.1), we get

$$D(R)\ \bar{v}_{pq}^{abcdefgh} = [Z_{pq}]^{abcdefgh}\ D(R)\bar{i}_{pq}^{abcdefgh}$$

or

$$\bar{v}_{pq}^{abcdefgh} = D^{-1}(R)[Z_{pq}]^{abcdefgh}\ D(R)\bar{i}_{pq}^{abcdefgh} \tag{3.1}$$

Comparing equation (2.2) and (3.1), we get

$$[Z_{pq}]^{abcdefgh} = D^{-1}(R)[Z_{pq}]^{abcdefgh}\ D(R)$$

Taking $R = C_8^1$, $D^{-1}(R) = D^{-1}(C_8^1) = D(C_8^7)$

$$[Z_{pq}]^{abcdefgh} = D(C_8^7)[Z_{pq}]^{abcdefgh}\ D(C_8^1) \tag{3.2}$$

Performing the matrix multiplications in Eqn.(3.2) on expanded form of $[Z_{pq}]$, we get

$$\begin{bmatrix}
z_{pq}^{aa} & z_{pq}^{ab} & z_{pq}^{ac} & z_{pq}^{ad} & z_{pq}^{ae} & z_{pq}^{af} & z_{pq}^{ag} & z_{pq}^{ah} \\
z_{pq}^{ba} & z_{pq}^{bb} & z_{pq}^{bc} & z_{pq}^{bd} & z_{pq}^{be} & z_{pq}^{bf} & z_{pq}^{bg} & z_{pq}^{bh} \\
z_{pq}^{ca} & z_{pq}^{cb} & z_{pq}^{cc} & z_{pq}^{cd} & z_{pq}^{ce} & z_{pq}^{cf} & z_{pq}^{cg} & z_{pq}^{ch} \\
z_{pq}^{da} & z_{pq}^{db} & z_{pq}^{dc} & z_{pq}^{dd} & z_{pq}^{de} & z_{pq}^{df} & z_{pq}^{dg} & z_{pq}^{dh} \\
z_{pq}^{ea} & z_{pq}^{eb} & z_{pq}^{ec} & z_{pq}^{ed} & z_{pq}^{ee} & z_{pq}^{ef} & z_{pq}^{eg} & z_{pq}^{eh} \\
z_{pq}^{fa} & z_{pq}^{fb} & z_{pq}^{fc} & z_{pq}^{fd} & z_{pq}^{fe} & z_{pq}^{ff} & z_{pq}^{fg} & z_{pq}^{fh} \\
z_{pq}^{ga} & z_{pq}^{gb} & z_{pq}^{gc} & z_{pq}^{gd} & z_{pq}^{ge} & z_{pq}^{gf} & z_{pq}^{gg} & z_{pq}^{gh} \\
z_{pq}^{ha} & z_{pq}^{hb} & z_{pq}^{hc} & z_{pq}^{hd} & z_{pq}^{he} & z_{pq}^{hf} & z_{pq}^{hg} & z_{pq}^{hh}
\end{bmatrix}$$

$$= \begin{bmatrix}
z_{pq}^{hh} & z_{pq}^{ha} & z_{pq}^{hb} & z_{pq}^{hc} & z_{pq}^{hd} & z_{pq}^{he} & z_{pq}^{hf} & z_{pq}^{hg} \\
z_{pq}^{ah} & z_{pq}^{aa} & z_{pq}^{ab} & z_{pq}^{ac} & z_{pq}^{ad} & z_{pq}^{ae} & z_{pq}^{af} & z_{pq}^{ag} \\
z_{pq}^{bh} & z_{pq}^{ba} & z_{pq}^{bb} & z_{pq}^{bc} & z_{pq}^{bd} & z_{pq}^{be} & z_{pq}^{bf} & z_{pq}^{bg} \\
z_{pq}^{ch} & z_{pq}^{ca} & z_{pq}^{cb} & z_{pq}^{cc} & z_{pq}^{cd} & z_{pq}^{ce} & z_{pq}^{cf} & z_{pq}^{cg} \\
z_{pq}^{dh} & z_{pq}^{da} & z_{pq}^{db} & z_{pq}^{dc} & z_{pq}^{dd} & z_{pq}^{de} & z_{pq}^{df} & z_{pq}^{dg} \\
z_{pq}^{eh} & z_{pq}^{ea} & z_{pq}^{eb} & z_{pq}^{ec} & z_{pq}^{ed} & z_{pq}^{ee} & z_{pq}^{ef} & z_{pq}^{eg} \\
z_{pq}^{fh} & z_{pq}^{fa} & z_{pq}^{fb} & z_{pq}^{fc} & z_{pq}^{fd} & z_{pq}^{fe} & z_{pq}^{ff} & z_{pq}^{fg} \\
z_{pq}^{gh} & z_{pq}^{ga} & z_{pq}^{gb} & z_{pq}^{gc} & z_{pq}^{gd} & z_{pq}^{ge} & z_{pq}^{gf} & z_{pq}^{gg}
\end{bmatrix}$$

Comparing two matrices, we get

$$z_{pq}^{aa} = z_{pq}^{bb} = z_{pq}^{cc} = z_{pq}^{dd} = z_{pq}^{ee} = z_{pq}^{ff} = z_{pq}^{gg} = z_{pq}^{hh} = z_{pq}^{s} \text{ (say)}$$

$$z_{pq}^{ab} = z_{pq}^{bc} = z_{pq}^{cd} = z_{pq}^{de} = z_{pq}^{ef} = z_{pq}^{fg} = z_{pq}^{gh} = z_{pq}^{ha} = z_{pq}^{m1} \text{ (say)}$$

$$z_{pq}^{ac} = z_{pq}^{ce} = z_{pq}^{eg} = z_{pq}^{ga} = z_{pq}^{hb} = z_{pq}^{bd} = z_{pq}^{df} = z_{pq}^{fh} = z_{pq}^{m2} \text{ (say)}$$

$$\vdots \quad \vdots \qquad\qquad\qquad\qquad \vdots$$

$$z_{pq}^{ah} = z_{pq}^{hg} = z_{pq}^{gf} = z_{pq}^{fe} = z_{pq}^{ed} = z_{pq}^{dc} = z_{pq}^{cb} = z_{pq}^{ba} = z_{pq}^{m7} \text{ (say)} \qquad (3.3)$$

From (3.3) we can see that, impedance matrix Z_{pq} is cyclic and of the form:

$$\begin{bmatrix}
z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} \\
z_{pq}^{m7} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} \\
z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} \\
z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} \\
z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} \\
z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} \\
z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{s} & z_{pq}^{m1} \\
z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{s}
\end{bmatrix} \qquad (3.4)$$

Now, the eigenvectors of permutation matrices (Eqn.2.3) are given by the following matrix A_c [11]:

$$A_c = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 \\ 1 & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a \\ 1 & a^3 & a^4 & a^5 & a^6 & a^7 & a & a^2 \\ 1 & a^4 & a^5 & a^6 & a^7 & a & a^2 & a^3 \\ 1 & a^5 & a^6 & a^7 & a & a^2 & a^3 & a^4 \\ 1 & a^6 & a^7 & a & a^2 & a^3 & a^4 & a^5 \\ 1 & a^7 & a & a^2 & a^3 & a^4 & a^5 & a^6 \end{bmatrix}$$

where $a = \sqrt[8]{1} = e^{j2\pi/8} = 1\angle 45^\circ = \frac{1}{\sqrt{2}}(1+j)$

$$a^2 = j\ , \quad a^3 = \frac{1}{\sqrt{2}}(-1+j) = -a^*, \quad a^4 = -1,$$

$$a^5 = -\frac{1}{\sqrt{2}}(1+j) = -a, \quad a^6 = -j = -a^2$$

and $a^7 = \frac{1}{\sqrt{2}}(1-j) = a^*$

So

$$A_c = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a & a^2 & -a^* & -1 & -a & -a^2 & a^* \\ 1 & a^2 & -a^* & -1 & -a & -a^2 & a^* & a \\ 1 & -a^* & -1 & -a & -a^2 & a^* & a & a^2 \\ 1 & -1 & -a & -a^2 & a^* & a & a^2 & -a^* \\ 1 & -a & -a^2 & a^* & a & a^2 & -a^* & -1 \\ 1 & -a^2 & a^* & a & a^2 & -a^* & -1 & -a \\ 1 & a^* & a & a^2 & -a^* & -1 & -a & -a^2 \end{bmatrix} \qquad (3.5)$$

The unitary matrix A_c, whose columns are eigenvector of D(R), will diagonalize each of the permutation matrices D(R) for $R = C_8^1, C_8^2, \ldots, C_8^7$, E, i.e.

$$A_c^{*T} D(R) A_c = \mathrm{Diag}\, D(R)$$

The coefficient matrix $[Z_{pq}]$ commutes with the permutation matrices and therefore the same unitary matrix will also diagonalize the coefficient matrix $[Z_{pq}]$. It is evident that the unitary matrix A_c is similar to the symmetrical component transformation matrix for a 3-phase system.

Now we will rederive the transformation for the eight phase cyclic symmetries using representation theory approach [1,5,6,8,14].

Representation Theory Approach:

A representation D(R) is said to be reducible if its matrices can be expressed as direct sums of matrices of smaller dimension obtained by a similarity transformation to each matrix of the group. Otherwise it is said to be irreducible. Let D(R) be a representation of a group S containing elements (R). Then, for a nonsingular matrix α, let $\bar{D}(R)$ be the similarity transformation of D(R) under α, i.e.

$$\bar{D}(R) = \alpha^{-1} D(R)\, \alpha$$

Thus a reducible representation can be converted to block diagonal form, called reduced out representation via a similarity transformation. It is generally understood that

transformation matrix α is unitary, i.e. $\alpha^{-1} = \alpha^{*T}$. The submatrix blocks on the diagonal of the reduced out representation are the irreducible representation of the group, i.e.

$$\bar{D}(R) = \alpha^{*T} D(R) \alpha = \begin{bmatrix} D^1(R) & & & 0 \\ & D^2(R) & & \\ & & \ddots & \\ 0 & & & \ddots \end{bmatrix} \tag{3.6}$$

where $D^1(R)$, $D^2(R)$ ... etc. are irreducible representations of the group not necessarily distinct. For any representation irreducible or not, an important set of invariants is that of its characters which are traces of the matrices of representations, i.e. characters of equivalent representations are the same.

We now give some important rules about irreducible representations and their characters which are the consequences of the orthogonality theorem (Appendix 2).

1. The sum of squares of the dimensions of the irreducible representations is equal to the order of the group, i.e. number of distinct elements of the group i.e.

$$\sum l_i^2 = l_1^2 + l_2^2 + \ldots = h \tag{3.7}$$

where h is order of the group and l_i is dimension of ith irreducible representation.

2. The sum of squares of characters in any irreducible representations is equal to the order of the group, i.e.

$$\sum_R (X^i(R))^2 = h \qquad (3.8)$$

where $X^i(R)$ is the character of ith irreducible representation.

3. The vectors whose components are the characters of two different irreducible representations are orthogonal, i.e.

$$\sum_R X^i(R)X^j(R)^* = 0 \quad \text{for } i \neq j \qquad (3.9)$$

4. In a given representation (reducible or irreducible) characters of all matrices belonging to the symmetry operation in the same class are identical, and

5. The number of irreducible representations in a group is equal to the number of classes in the group.

As already mentioned, any reducible matrix can be reduced to a similar matrix consisting of blocks on the diagonal via a similarity transformation α. Since the character, i.e. trace of matrix is not changed by similarity transformation, we have

$$X(R) = \sum_j a_j \; X^j(R)$$

where X(R) is character of ith reducible representation, $X^j(R)$ is that of jth irreducible representation, aj is number of times the irreducible representation. $D^j(R)$ appears in the block diagonal of reduced out representation $\bar{D}(R)$ of D(R).

From above we get

$$a_j = \frac{1}{h} \sum \chi(R)\ \chi^j(R) \tag{3.10}$$

and $\bar{D}(R)$ will be of the form

$$\bar{D}(R) = \alpha^{*T}\ D(R)\ \alpha$$

$$= \begin{bmatrix} D^1(R) & & & & & & & & \\ & D^2(R) & & & & & & & \\ & & \ddots & & & & & & \\ & & & D^j(R) & & & & & \\ & & & & D^j(R) & & & & \\ & & & & & \ddots & & & \\ & & & & & & D^j(R) & & \\ & & & & & & & \ddots & \\ & & & & & & & & \ddots \end{bmatrix} \tag{3.11}$$

Now, we consider block diagonalization of matrices which commute with $D(R)$. The similarity transformation α which block diagonalizes $D(R)$ has a significant property [12,13,1] that it also diagonalizes matrices which commute with $D(R)$. Let α be the transformation which reduces $D(R)$ to the block diagonal form given by equation (3.11), in which jth irreducible representation is of dimension l_j and repeats a_j times in the block diagonal. Let A be the matrix which commutes with $D(R)$, i.e.

$$A\ D(R) = D(R)\ A$$

Then, α also transforms A into a block diagonal form $\bar{A}$.

$$\bar{A} = \alpha^{*T} A \alpha = \begin{bmatrix} \bar{A}_1 & & & & & & 0 \\ & \bar{A}_2 & & & & & \\ & & \ddots & & & & \\ & & & \bar{A}_j & & & \\ & & & & \ddots & & \\ & & & \underbrace{}_{l_j} & & \bar{A}_j & \\ & & & & & & \ddots \\ 0 & & & & & & \bar{A}_k \end{bmatrix}$$

where A_j repeats l_j times on the diagonal of $\bar{A}$, and is of dimension a_j corresponding to the irreducible representation $D^j(R)$.

We have shown in Chapter 2 that permutation matrices (Eqn. 2.3) representing rotation 1 symmetries of a 8-phase power system network form a cyclic group of order 8 and each element of group is in a separate class. Therefore, the number of classes in group C_8 will be equal to 8 and the number of irreducible representations which are equal to the number of classes, will also be equal to 8. Let $l_1, l_2, \ldots, l_8$ be the dimensions of these irreducible representations, then from Eqn.(3.7), we get,

$$\sum_{i=1}^{8} l_i^2 = l_1^2 + l_2^2 + \ldots + l_8^2 = 8$$

i.e. $$l_1 = l_2 = l_3 = l_4 = l_5 = l_6 = l_7 = l_8 = 1$$

Therefore, all the irreducible representations have the dimension equal to 1. Likewise, from Eqn.(3.8),

$$\sum X_i^2(R) = 8$$

and hence the characters $X^i(R)$ are each of unity modulus. Using standard computational techniques based on the orthogonality theorem (Appendix 2), one then gets the character table shown below:

	C_8	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7
	$X^1(R)$	1	1	1	1	1	1	1	1
	$X^2(R)$	1	a	a^2	a^3	a^4	a^5	a^6	a^7
	$X^3(R)$	1	a^2	a^3	a^4	a^5	a^6	a^7	a
	$X^4(R)$	1	a^3	a^4	a^5	a^6	a^7	a	a^2
	$X^5(R)$	1	a^4	a^5	a^6	a^7	a	a^2	a^3
	$X^6(R)$	1	a^5	a^6	a^7	a	a^2	a^3	a^4
	$X^7(R)$	1	a^6	a^7	a	a^2	a^3	a^4	a^5
	$X^8(R)$	1	a^7	a	a^2	a^3	a^4	a^5	a^6
Red. Rep.	$X(R)$	8	0	0	0	0	0	0	0

(3.13)

CHARACTER TABLE

where $a = e^{j2\pi/8} = \frac{1}{\sqrt{2}}(1+j)$

Evidently in this case, the character table itself gives the irreducible representation of the group,

C_8	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7
$D^1(R)$	1	1	1	1	1	1	1	1
$D^2(R)$	1	a	a^2	a^3	a^4	a^5	a^6	a^7
$D^3(R)$	1	a^2	a^3	a^4	a^5	a^6	a^7	a
$D^4(R)$	1	a^3	a^4	a^5	a^6	a^7	a	a^2
$D^5(R)$	1	a^4	a^5	a^6	a^7	a	a^2	a^3
$D^6(R)$	1	a^5	a^6	a^7	a	a^2	a^3	a^4
$D^7(R)$	1	a^6	a^7	a	a^2	a^3	a^4	a^5
$D^8(R)$	1	a^7	a	a^2	a^3	a^4	a^5	a^6

(3.14)

IRREDUCIBLE REPRESENTATION

The number of times each of these irreducible representations, $D^i(R)$ appear at the diagonal of the reduced out representation $\bar{D}(R)$ is from Eqn.(3.10),

$$a_i = \frac{1}{h}\sum_R X^i(R)X(R)$$

$$a_1 = \frac{1}{8}\sum_R X^1(R)X(R) = \frac{1}{8}(\ 1(8) + 1(0)+1(0)+1(0)+1(0)+1(0)+1(0) + 1(0)\ ) = 1$$

$$a_2 = \frac{1}{8}\sum_R X^2(R)X(R) = \frac{1}{8}(\ 1(8)+a(0)+a^2(0)+a^3(0)+a^4(0)+a^5(0) + a^6(0)+a^7(0)\ ) = 1$$

Similarly, we get

$$a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = 1$$

Therefore, $\overline{D}(R)$ is of the form:

$$\overline{D}(R) = \begin{bmatrix} D^1(R) & & & & & & & \\ & D^2(R) & & & & & 0 & \\ & & D^3(R) & & & & & \\ & & & D^4(R) & & & & \\ & & & & D^5(R) & & & \\ & 0 & & & & D^6(R) & & \\ & & & & & & D^7(R) & \\ & & & & & & & D^8(R) \end{bmatrix} \tag{3.15}$$

In order to determine the similarity transformation, $\mathcal{L}$, the following matrix based upon the orthogonality theorem is constructed[12, 13, 14, 16]i.e.

$$G_i^j = \sum_R D^j(R)_{ii}\; D(R) \tag{3.16}$$

where $D^j(R)_{ii}$ is the diagonal element of the irreducible representation $D^j(R)$ for $j = 1,2,\ldots,8$ counting i once for every appearance of reducible representation D(R). Then, the basis vector α_{mnp}, where α_{mnp} denotes the pth linear independent vector corresponding to mth irreducible representation appearing for nth time, is constructed by scanning matrices G_i^j from left to right and picking first the linearly independent columns of G_1^1, then of G_1^2, G_1^3, G_1^4, G_1^5, G_1^6, G_1^7 and G_1^8. Now from Eqn.(3.16) we have

$$\begin{aligned} G_1^1 = \sum_R D^1(R)_{11}\; D(R) = {} & D^1(E)_{11}\; D(E)+D^1(R_1)_{11}D(R_1)+D^1(R_2)_{11}D(R_2) \\ & + D^1(R_3)_{11}D(R_3)+D^1(R_4)_{11}D(R_4)+D^1(R_5)_{11}D(R_5) \\ & +D^1(R_6)_{11}D(R_6)+D^1(R_7)_{11}D(R_7) \end{aligned}$$

where $R_1 = C_8^1$, $R_2 = C_8^2$, $R_3 = C_8^3$, $R_4 = C_8^4$, $R_5 = C_8^5$, $R_6 = C_8^6$,

and $R_7 = C_8^7$

$$= 1\ D(E) + 1\ D(R_1) + 1\ D(R_2) + 1\ D(R_3) + 1\ D(R_4) + 1\ D(R_5) + 1\ D(R_6) + 1\ D(R_7)$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$G_1^2 = \sum_R D^2(R)_{11}\ D(R)$$

$$= 1\ D(E) + a\ D(R_1) + a^2 D(R_2) + a^3 D(R_3) + a^4 D(R_4) + a^5 D(R_5) + a^6 D(R_6) + a^7 D(R_7)$$

$$= \begin{bmatrix} 1 & a^7 & a^6 & a^5 & a^4 & a^3 & a^2 & a \\ a & 1 & a^7 & a^6 & a^5 & a^4 & a^3 & a^2 \\ a^2 & a & 1 & a^7 & a^6 & a^5 & a^4 & a^3 \\ a^3 & a^2 & a & 1 & a^7 & a^6 & a^5 & a^4 \\ a^4 & a^3 & a^2 & a & 1 & a^7 & a^6 & a^5 \\ a^5 & a^4 & a^3 & a^2 & a & 1 & a^7 & a^6 \\ a^6 & a^5 & a^4 & a^3 & a^2 & a & 1 & a^7 \\ a^7 & a^6 & a^5 & a^4 & a^3 & a^2 & a & 1 \end{bmatrix}$$

$$G_1^3 = \sum_R D^3(R)_{11}\ D(R) = 1\ D(E) + a^2 D(R_1) + a^3 D(R_2) + a^4 D(R_3) + a^5 D(R_4) + a^6\ D(R_5) + a^7 D(R_6) + a\ D(R_7)$$

$$= \begin{bmatrix} 1 & a & a^7 & a^6 & a^5 & a^4 & a^3 & a^2 \\ a^2 & 1 & a & a^7 & a^6 & a^5 & a^4 & a^3 \\ a^3 & a^2 & 1 & a & a^7 & a^6 & a^5 & a^4 \\ a^4 & a^3 & a^2 & 1 & a & a^7 & a^6 & a^5 \\ a^5 & a^4 & a^3 & a^2 & 1 & a & a^7 & a^6 \\ a^6 & a^5 & a^4 & a^3 & a^2 & 1 & a & a^7 \\ a^7 & a^6 & a^5 & a^4 & a^3 & a^2 & 1 & a \\ a & a^7 & a^6 & a^5 & a^4 & a^3 & a^2 & 1 \end{bmatrix}$$

$$G_1^4 = \sum_R D^4(R)_{11} D(R) = 1\ D(E) + a^3 D(R_1) + a^4 D(R_2) + a^5 D(R_3) + a^6 D(R_4) + a^7 D(R_5) + a D(R_6)\ + a^2 D(R_7)$$

$$= \begin{bmatrix} 1 & a^2 & a & a^7 & a^6 & a^5 & a^4 & a^3 \\ a^3 & 1 & a^2 & a & a^7 & a^6 & a^5 & a^4 \\ a^4 & a^3 & 1 & a^2 & a & a^7 & a^6 & a^5 \\ a^5 & a^4 & a^3 & 1 & a^2 & a & a^7 & a^6 \\ a^6 & a^5 & a^4 & a^3 & 1 & a^2 & a & a^7 \\ a^7 & a^6 & a^5 & a^4 & a^3 & 1 & a^2 & a \\ a & a^7 & a^6 & a^5 & a^4 & a^3 & 1 & a^2 \\ a^2 & a & a^7 & a^6 & a^5 & a^4 & a^3 & 1 \end{bmatrix}$$

$$G_1^5 = \sum_R D^5(R)_{11} D(R) = 1\ D(E) + a^4 D(R_1) + a^5 D(R_2) + a^6 D(R_3) + a^7 D(R_4) + a D(R_5) + a^2 D(R_6) + a^3 D(R_7)$$

$$
= \begin{bmatrix}
1 & a^3 & a^2 & a & a^7 & a^6 & a^5 & a^4 \\
a^4 & 1 & a^3 & a^2 & a & a^7 & a^6 & a^5 \\
a^5 & a^4 & 1 & a^3 & a^2 & a & a^7 & a^6 \\
a^6 & a^5 & a^4 & 1 & a^3 & a^2 & a & a^7 \\
a^7 & a^6 & a^5 & a^4 & 1 & a^3 & a^2 & a \\
a & a^7 & a^6 & a^5 & a^4 & 1 & a^3 & a^2 \\
a^2 & a & a^7 & a^6 & a^5 & a^4 & 1 & a^3 \\
a^3 & a^2 & a & a^7 & a^6 & a^5 & a^4 & 1
\end{bmatrix}
$$

$$
G_1^6 = \sum_R D^6(R)_{11} D(R) = 1\ D(E) + a^5 D(R_1) + a^6 D(R_2) + a^7 D(R_3) + a\ D(R_4) + a^2 D(R_5) + a^3 D(R_6) + a^4 D(R_7)
$$

$$
= \begin{bmatrix}
1 & a^4 & a^3 & a^2 & a & a^7 & a^6 & a^5 \\
a^5 & 1 & a^4 & a^3 & a^2 & a & a^7 & a^6 \\
a^6 & a^5 & 1 & a^4 & a^3 & a^2 & a & a^7 \\
a^7 & a^6 & a^5 & 1 & a^4 & a^3 & a^2 & a \\
a & a^7 & a^6 & a^5 & 1 & a^4 & a^3 & a^2 \\
a^2 & a & a^7 & a^6 & a^5 & 1 & a^4 & a^3 \\
a^3 & a^2 & a & a^7 & a^6 & a^5 & 1 & a^4 \\
a^4 & a^3 & a^2 & a & a^7 & a^6 & a^5 & 1
\end{bmatrix}
$$

$$
G_1^7 = \sum_R D^7(R)_{11}\ D(R) = 1\ D(E) + a^6 D(R_1) + a^7 D(R_2) + a\ D(R_3) + a^2 D(R_4) + a^3 D(R_5) + a^4 D(R_6) + a^5 D(R_7)
$$

$$= \begin{bmatrix} 1 & a^5 & a^4 & a^3 & a^2 & a & a^7 & a^6 \\ a^6 & 1 & a^5 & a^4 & a^3 & a^2 & a & a^7 \\ a^7 & a^6 & 1 & a^5 & a^4 & a^3 & a^2 & a \\ a & a^7 & a^6 & 1 & a^5 & a^4 & a^3 & a^2 \\ a^2 & a & a^7 & a^6 & 1 & a^5 & a^4 & a^3 \\ a^3 & a^2 & a & a^7 & a^6 & 1 & a^5 & a^4 \\ a^4 & a^3 & a^2 & a & a^7 & a^6 & 1 & a^5 \\ a^5 & a^4 & a^3 & a^2 & a & a^7 & a^6 & 1 \end{bmatrix}$$

and $G_1^8 = \sum_R D^8(R)_{11} D(R) = 1\, D(E) + a^7 D(R_1) + a\, D(R_2) + a^2 D(R_3) + a^3 D(R_4)$
$+ a^4 D(R_5) + a^5 D(R_6) + a^6 D(R_7)$

$$= \begin{bmatrix} 1 & a^6 & a^5 & a^4 & a^3 & a^2 & a & a^7 \\ a^7 & 1 & a^6 & a^5 & a^4 & a^3 & a^2 & a \\ a & a^7 & 1 & a^6 & a^5 & a^4 & a^3 & a^2 \\ a^2 & a & a^7 & 1 & a^6 & a^5 & a^4 & a^3 \\ a^3 & a^2 & a & a^7 & 1 & a^6 & a^5 & a^4 \\ a^4 & a^3 & a^2 & a & a^7 & 1 & a^6 & a^5 \\ a^5 & a^4 & a^3 & a^2 & a & a^7 & 1 & a^6 \\ a^6 & a^5 & a^4 & a^3 & a^2 & a & a^7 & 1 \end{bmatrix}.$$

Now, the basis vector α_{mnp} after normalization to unity will be as shown

$$\alpha_{111} = \frac{1}{\sqrt{8}} [\,1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\,]^T$$

$$\alpha_{211} = \frac{1}{\sqrt{8}} [\,1 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5 \quad a^6 \quad a^7\,]^T$$

$$\alpha_{311} = \frac{1}{\sqrt{8}}[\,1 \quad a^2 \quad a^3 \quad a^4 \quad a^5 \quad a^6 \quad a^7 \quad a\,]^T$$

$$\alpha_{411} = \frac{1}{\sqrt{8}}[\,1 \quad a^3 \quad a^4 \quad a^5 \quad a^6 \quad a^7 \quad a \quad a^2\,]^T$$

$$\alpha_{511} = \frac{1}{\sqrt{8}}[\,1 \quad a^4 \quad a^5 \quad a^6 \quad a^7 \quad a \quad a^2 \quad a^3\,]^T$$

$$\alpha_{611} = \frac{1}{\sqrt{8}}[\,1 \quad a^5 \quad a^6 \quad a^7 \quad a \quad a^2 \quad a^3 \quad a^4\,]^T$$

$$\alpha_{711} = \frac{1}{\sqrt{8}}[\,1 \quad a^6 \quad a^7 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5\,]^T$$

and

$$\alpha_{811} = \frac{1}{\sqrt{8}}[\,1 \quad a^7 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5 \quad a^6\,]^T$$

Thus the matrix $\alpha = A_c$ comes out to be

$$[A_c] = [\alpha_{111} \quad \alpha_{211} \quad \alpha_{311} \quad \alpha_{411} \quad \alpha_{511} \quad \alpha_{611} \quad \alpha_{711} \quad \alpha_{811}]$$

$$D^1(R) \quad D^2(R) \quad D^3(R) \quad D^4(R) \quad D^5(R) \quad D^6(R) \quad D^7(R) \quad D^8(R)$$

$$= \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 \\ 1 & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a \\ 1 & a^3 & a^4 & a^5 & a^6 & a^7 & a & a^2 \\ 1 & a^4 & a^5 & a^6 & a^7 & a & a^2 & a^3 \\ 1 & a^5 & a^6 & a^7 & a & a^2 & a^3 & a^4 \\ 1 & a^6 & a^7 & a & a^2 & a^3 & a^4 & a^5 \\ 1 & a^7 & a & a^2 & a^3 & a^4 & a^5 & a^6 \end{bmatrix} \qquad (3.16)$$

We can see that this matrix is same as the one given in Eqn.(3.5).

It can be verified that $A_c^{*T} A_c = I$ hence $A_c^{-1} = A_c^{*T}$ i.e. A_c is unitary matrix. Now, from Eqn.(2.2)

$$\bar{v}_{pq}^{phase} = [Z_{pq}]^{phase} \; \bar{i}_{pq}^{phase}$$

We know that

$$\bar{v}_{pq}^{phase} = A_c \; \bar{v}_{pq}^{comp.} \quad \text{and} \quad \bar{i}_{pq}^{phase} = A_c \; \bar{i}_{pq}^{comp.}$$

So
$$A_c \; \bar{v}_{pq}^{comp} = [Z_{pq}]^{phase} \; A_c \; \bar{i}_{pq}^{comp}$$

or
$$\bar{v}_{pq}^{comp} = A_c^{*T} \; [Z_{pq}]^{phase} \; A_c \; \bar{i}_{pq}^{comp} = [Z_{pq}]^{comp} \; \bar{i}_{pq}^{comp}$$

Hence, $$[Z_{pq}]^{comp} = A_c^{*T} \; [Z_{pq}]^{phase} \; A_c \qquad (3.17)$$

Substituting values of A_c^{*T} and A_c from eqn.(3.5) and $[Z_{pq}]^{phase}$ from eqn.(3.4) in eqn.(3.17), we get

$$[Z_{pq}]^{0,1,2,3,4,5,6,7}_{comp}$$

$$= \begin{bmatrix} z_{pq}^{s}+z_{pq}^{m1}+z_{pq}^{m2}+z_{pq}^{m3}+z_{pq}^{m4}+z_{pq}^{m5}+z_{pq}^{m6}+z_{pq}^{m7} \\ z_{pq}^{s}+az_{pq}^{m1}+a^2 z_{pq}^{m2}-a^* z_{pq}^{m3}-z_{pq}^{m4}-az_{pq}^{m5}-a^2 z_{pq}^{m6}+a^* z_{pq}^{m7} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad (3.18)$$

Therefore,

$$z_{pq}^{o} = z_{pq}^{s}+z_{pq}^{m1}+z_{pq}^{m2}+z_{pq}^{m3}+z_{pq}^{m4}+z_{pq}^{m5}+z_{pq}^{m6}+z_{pq}^{m7}$$

$$z_{pq}^{1} = z_{pq}^{s}+az_{pq}^{m1}+a^2 z_{pq}^{m2}-a^* z_{pq}^{m3}-z_{pq}^{m4}-az_{pq}^{m5}-a^2 z_{pq}^{m6}+a^* z_{pq}^{m7}$$

and so on.

Here z^{0}_{pq} is the zero sequence impedance and z^{1}_{pq}, z^{2}_{pq} ... etc. are the first, second.... sequence impedances respectively.

Proposition: The transformation matrix A_c which diagonalizes the coefficient matrix of 8-phase rotating elements is a linear power invariant transformation matrix with complex elements similar to the symmetrical components.

3.2 8-PHASE STATIONARY ELEMENTS

In the previous chapter we have seen that 8-phase network which possesses reflection symmetries in addition to rotation symmetries, commonly known as stationary elements (typical example is that of 8-phase transposed transmission line) are symmetric under symmetry operations viz. C^1_8, C^2_8, C^3_8, C^4_8, C^5_8, C^6_8, C^7_8, E, δ'_a, δ'_b, δ'_c, δ'_d, δ'_e, δ'_f, δ'_g and δ'_h. In the previous section we have seen that impedance matrix for a network possessing rotational symmetries is cyclic (Eqn.(3.4)). For the network possessing both rotational as well as reflection symmetries, applying Eqn.(3.1) for symmetry operation of reflection, we get for $R = \delta'_a$.

$$[Z_{pq}]^{abcdefgh} = D^{-1}(\delta'_a)[Z_{pq}]^{abcdefgh}\, D(\delta'_a)$$

We know that $D^{-1}(\delta'_a) = D(\delta'_a)$. Now substituting the values of $D(\delta'_a)$ from Eqn.(2.6) and $[Z_{pq}]$ from Eqn.(3.4) and then comparing terms of matrices on the two sides, we get that $[Z_{pq}]$ is of the form:

$$[Z_{pq}]^{abcdefgh} = \begin{bmatrix} z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\ z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\ z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\ z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\ z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\ z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} \\ z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} \\ z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} \end{bmatrix} \quad (3.20)$$

As shown in Chapter 2, these sixteen symmetry operation form a group C_{8v} which are represented by permutation matrices given in equation (2.3) and equation (2.6). The order of the group C_{8v} is sixteen, but there are only seven classes viz. $[D(E)]$, $[D(C_8^1), D(C_8^7)]$, $[D(C_8^2), D(C_8^6)]$, $[D(C_8^3), D(C_8^5)]$, $[D(C_8^4)]$, $[D(\delta'_a), D(\delta'_b), D(\delta'_c), D(\delta'_d)]$ and $[D(\delta'_e), D(\delta'_f), D(\delta'_g), D(\delta'_h)]$. Therefore, the number of irreducible representations is also seven. Let l_1, l_2, l_3, l_4, l_5, l_6 and l_7 be the dimensions of these irreducible representations. Then from Eqn.(3.7), we have

$$\sum_{i=1}^{7} l_i^2 = l_1^2 + l_2^2 + \dots + l_7^2 = h = 16$$

The solution of this equation is $l_1 = l_2 = l_3 = l_4 = 1$ and $l_5 = l_6 = l_7 = 2$. From this we conclude that there are four irreducible representations viz. $D^1(R)$, $D^2(R)$, $D^3(R)$ and

$D^4(R)$ of dimension 1 each and three irreducible representations viz. $D^5(R)$, $D^6(R)$ and $D^7(R)$ of dimension 2 each. Since, from equation (3.8)

$$\sum_R \chi_i^2(R) = h = 16$$

the characters of each of sixteen elements of the first four irreducible representations whose dimension is 1, is 1 or −1, but that of the last three representations whose dimension is 2, is 2, −2 or 0. Hence the character table is

C_{8v}	E	C_8^1	C_8^2	C_8^3	C_8^4	C_8^5	C_8^6	C_8^7	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f	δ'_g	δ'_h
$\chi^1(R)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi^2(R)$	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
$\chi^3(R)$	1	−1	1	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1
$\chi^4(R)$	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1
$\chi^5(R)$	2	0	−2	0	−2	0	2	0	0	0	0	0	0	0	0	0
$\chi^6(R)$	2	0	2	0	−2	0	−2	0	0	0	0	0	0	0	0	0
$\chi^7(R)$	2	0	−2	0	2	0	−2	0	0	0	0	0	0	0	0	0
Red.Rep. $\chi(R)$	8	0	0	0	0	0	0	0	2	2	2	2	0	0	0	0

(3.21)

It is evident that the irreducible representations of $D^1(R)$, $D^2(R)$, $D^3(R)$ and $D^4(R)$ whose dimensions are 1, are the same as their characters but $D^5(R)$, $D^6(R)$ and $D^7(R)$ are not. Using the orthogonality theorem (Appendix 2) and its consequences the irreducible representation of the group comes out to be:

The number of times the irreducible representations $D^{i}(R)$ appear at the diagonal of $\overline{D}(R)$ is determined as follows:

$1 \ldots 1 \ldots$ $\quad 1\,(1(8)+1(0)+1(0)+1(0)+1(0)+1(0)+1(0)+1(0))$

$= 1$

$0)$

$) = \ldots$

E	C
1	[illegible]
1	[illegible]
1	− [illegible]
1	− [illegible]
$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}0 & \ldots\\-1 & \ldots\end{bmatrix}$
$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}0 & \ldots\\1 & \ldots\end{bmatrix}$
$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}0 & \ldots\\1 & \ldots\end{bmatrix}$

The number of times the irreducible representations $D^i(R)$ appear at the diagonal of $\overline{D}(R)$ is determined as follows:

$$a_1 = \frac{1}{h}\sum_R X^1(R)X(R) = \frac{1}{16}(1(8)+1(0)+1(0)+1(0)+1(0)+1(0)+1(0)+1(0)$$
$$+1(2)+1(2)+1(2)+1(2)+1(0)+1(0)+1(0)+1(0)) = 1$$

$$a_2 = \frac{1}{h}\sum_R X^2(R)X(R) = \frac{1}{16}(1(8)+1(0)+1(0)+1(0)+1(0)+1(0)+1(0)+1(0)$$
$$-1(2)-1(2)-1(2)-1(2)-1(0)-1(0)-1(0)-1(0)) = 0$$

Similarly

$$a_3 = \frac{1}{h}\sum_R X^3(R)X(R) = \frac{1}{16}(1(8)+1(2)+1(2)+1(2)+1(2)) = 1$$

$$a_4 = \frac{1}{h}\sum_R X^4(R)X(R) = \frac{1}{16}(1(8)-1(2)-1(2)-1(2)-1(2)) = 0$$

$$a_5 = \frac{1}{h}\sum_R X^5(R)X(R) = \frac{1}{16}(2(8)+0) = 1$$

$$a_6 = \frac{1}{h}\sum_R X^6(R)X(R) = \frac{1}{16}(2(8)+0) = 1$$

$$a_7 = \frac{1}{h}\sum_R X^7(R)X(R) = \frac{1}{16}(2(8)+0) = 1$$

Let transformation matrix α be designated by A_r, then

$$\overline{D}(R) = A_r^{*T} D(R)A_r = \begin{bmatrix} D^1(R) & & & & \\ & D^3(R) & & 0 & \\ & & D^5(R) & & \\ & 0 & & D^6(R) & \\ & & & & D^7(R) \end{bmatrix} \qquad (3.23)$$

Now we determine the matrix G_i^j (following the same procedure as in previous section)as follows:

$$G_1^1 = \sum_R D^1(R)_{11}\ D(R) \quad \text{for } R = E,\ C_8^1,\ C_8^2,\ \ldots,\ C_8^7 \text{ and } \delta_a',\ \delta_b',\ldots,\delta_h'$$

$$= 1D(E)+1D(C_8^1)+1D(C_8^2)+1D(C_8^3)+1D(C_8^4)+1D(C_8^5)+1D(C_8^6)+1D(C_8^7)+1D(\delta_a')$$
$$+1D(\delta_b')+1D(\delta_c')+1D(\delta_d')+1D(\delta_e')+1D(\delta_f')+1D(\delta_g')+1D(\delta_h')$$

$$= \begin{bmatrix} 2&2&2&2&2&2&2&2\\ 2&2&2&2&2&2&2&2\\ 2&2&2&2&2&2&2&2\\ 2&2&2&2&2&2&2&2\\ 2&2&2&2&2&2&2&2\\ 2&2&2&2&2&2&2&2\\ 2&2&2&2&2&2&2&2\\ 2&2&2&2&2&2&2&2 \end{bmatrix}$$

$$G_1^2 = \sum_R D^2(R)_{11}D(R) = 1D(E)+1D(C_8^1)+1D(C_8^2)+1D(C_8^3)+1D(C_8^4)+1D(C_8^5)$$
$$+1D(C_8^6)+1D(C_8^7)-1D(\delta_a')-1D(\delta_b')-1D(\delta_c')-1D(\delta_d')$$
$$-1D(\delta_e')-1D(\delta_f')-1D(\delta_g')-1D(\delta_h') = [0]$$

$$G_1^3 = \sum_R D^3(R)_{11}D(R) = 1D(E)-1D(C_8^1)+1D(C_8^2)-1D(C_8^3)+1D(C_8^4)-1D(C_8^5)$$
$$+1D(C_8^6)-1D(C_8^7)+1D(\delta_a')+1D(\delta_b')+1D(\delta_c')+1D(\delta_d')$$
$$-1D(\delta_e')-1D(\delta_f')-1D(\delta_g')-1D(\delta_h')$$

$$= \begin{bmatrix} 2 & -2 & 2 & -2 & 2 & -2 & 2 & -2 \\ -2 & 2 & -2 & 2 & -2 & 2 & -2 & 2 \\ 2 & -2 & 2 & -2 & 2 & -2 & 2 & -2 \\ -2 & 2 & -2 & 2 & -2 & 2 & -2 & 2 \\ 2 & -2 & 2 & -2 & 2 & -2 & 2 & -2 \\ -2 & 2 & -2 & 2 & -2 & 2 & -2 & 2 \\ 2 & -2 & 2 & -2 & 2 & -2 & 2 & -2 \\ -2 & 2 & -2 & 2 & -2 & 2 & -2 & 2 \end{bmatrix}$$

$$G_1^4 = \sum_R D^4(R)_{11} D(R) = 1D(E) - 1D(C_8^1) + 1D(C_8^2) - 1D(C_8^3) + 1D(C_8^4) - 1D(C_8^5)$$
$$+ 1D(C_8^6) - 1D(C_8^7) - 1D(\delta_a') - 1D(\delta_b') - 1D(\delta_c') - 1D(\delta_d')$$
$$+ 1D(\delta_e') + 1D(\delta_f') + 1D(\delta_g') + 1D(\delta_h') = [0]$$

$$G_1^5 = \sum_R D^5(R)_{11} D(R) = 1D(E) - 1D(C_8^2) - 1D(C_8^4) + 1D(C_8^6) + 1D(\delta_a') + 1D(\delta_b')$$
$$- 1D(\delta_c') - 1D(\delta_d')$$

$$= \begin{bmatrix} 2 & 0 & 2 & 0 & -2 & 0 & -2 & 0 \\ 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 \\ -2 & 0 & -2 & 0 & 2 & 0 & 2 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 \end{bmatrix}$$

$$G_2^5 = \sum_R D^5(R)_{22} D(R)$$

$$= 1D(E) - 1D(C_8^2) - 1D(C_8^4) + 1D(C_8^6) - 1D(\delta_a') - 1D(\delta_b') + 1D(\delta_c') + 1D(\delta_d')$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & -2 \\ -2 & 0 & 2 & 0 & 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 & 2 \\ 2 & 0 & -2 & 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$G_1^6 = \sum_R D^6(R)_{11} D(R)$$

$$= 1D(E)+1D(C_8^2)-1D(C_8^4)-1D(C_8^6)+1D(\delta'_e)+1D(\delta'_f)-1D(\delta'_g)-1D(\delta'_h)$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

$$G_2^6 = \sum_R D^6(R)_{22} D(R)$$

$$= 1D(E)+1D(C_8^2)-1D(C_8^4)-1D(C_8^6)-1D(\delta'_e)-1D(\delta'_f)+1D(\delta'_g)+1D(\delta'_h)$$

$$= \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\ -1 & 2 & 1 & -2 & -1 & 2 & 1 & -2 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 1 & -2 & -1 & 2 & 1 & -2 & -1 & 2 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\ -1 & 2 & 1 & -2 & -1 & 2 & 1 & -2 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 1 & -2 & -1 & 2 & 1 & -2 & -1 & 2 \end{bmatrix}$$

The basis vector α_{mnp} after normalization to unity comes out to be

$$\alpha_{111} = \frac{1}{\sqrt{8}}[1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]^T$$

$$\alpha_{311} = \frac{1}{\sqrt{8}}[1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1]^T$$

$$\alpha_{511} = \frac{1}{\sqrt{8}}[2 \quad 0 \quad 0 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0]^T$$

$$\alpha_{521} = \frac{1}{\sqrt{8}}[0 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0 \quad 2 \quad 0]^T$$

$$\alpha_{611} = \frac{1}{\sqrt{8}}[1 \quad 1 \quad 1 \quad 1 \quad -1 \quad -1 \quad -1 \quad -1]^T$$

$$\alpha_{621} = \frac{1}{\sqrt{8}}[1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1]^T$$

$$\alpha_{711} = \frac{1}{\sqrt{8}}[\sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0]^T$$

$$\alpha_{721} = \frac{1}{\sqrt{8}}[0 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0 \quad \sqrt{2}]^T$$

Therefore, the matrix $\alpha = A_r$

$$= \begin{array}{cccccccc} \alpha_{111} & \alpha_{311} & \alpha_{511} & \alpha_{521} & \alpha_{611} & \alpha_{621} & \alpha_{711} & \alpha_{721} \\ D^1(R) & D^3(R) & \multicolumn{2}{c}{D^5(R)} & \multicolumn{2}{c}{D^6(R)} & \multicolumn{2}{c}{D^7(R)} \end{array}$$

$$= \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 2 & 0 & 1 & 1 & \sqrt{2} & 0 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & -\sqrt{2} \\ 1 & 1 & 0 & -2 & 1 & 1 & -\sqrt{2} & 0 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & \sqrt{2} \\ 1 & 1 & -2 & 0 & -1 & -1 & \sqrt{2} & 0 \\ 1 & -1 & 0 & 0 & -1 & 1 & 0 & -\sqrt{2} \\ 1 & 1 & 0 & 2 & -1 & -1 & -\sqrt{2} & 0 \\ 1 & -1 & 0 & 0 & -1 & 1 & 0 & \sqrt{2} \end{bmatrix} \qquad (3.24)$$

It can be verified that $A_r^T A_r = I = A_r^{-1} A_r$, and therefore the transformation matrix A_r is orthogonal. Now, from Eqn. (3.17)

$$[Z_{pq}]^{0,1,2,3,4,5,6,7}_{comp} = A_r^T [Z_{pq}]^{abcdefgh}_{phase} A_r$$

Substituting values for A_r and $[Z_{pq}]_{phase}$ from Eqn. (3.24) and (3.20) respectively, we get the impedance matrix in the component form:

$$[Z_{pq}]^{0,1,2,3,4,5,6,7}_{comp} = \begin{bmatrix} z^s_{pq}+7z^m_{pq} & & & & & & & \\ & z^s_{pq}-z^m_{pq} & & & & & 0 & \\ & & z^s_{pq}-z^m_{pq} & & & & & \\ & & & z^s_{pq}-z^m_{pq} & & & & \\ & & & & z^s_{pq}-z^m_{pq} & & & \\ & & & & & z^s_{pq}-z^m_{pq} & & \\ & 0 & & & & & z^s_{pq}-z^m_{pq} & \\ & & & & & & & z^s_{pq}-z^m_{pq} \end{bmatrix} \quad (3.25)$$

From this it is clear that the transformation matrix A_r diagonalizes the coefficient matrix $[Z_{pq}]$ of 8-phase stationary elements. It is to be noted here that the matrix A_c derived earlier can also diagonalize $[Z_{pq}]$ since stationary elements also possess rotational symmetries in addition to reflection ones. The diagonal elements of $[Z_{pq}]_{comp}$ are the sequence impedances in this case, more specifically, the zero sequence impedance $z^o_{pq}=z^s_{pq}+7z^m_{pq}$ and 1st, 2nd, 3rd, 4th, 5th, 6th and 7th sequence impedances are all equal to $z^s_{pq}-z^m_{pq}$.

Complex Power

The complex power in the 8-phase stationary element p-q

$$S_{pq} = P_{pq}-jQ_{pq} = \bar{v}^{abcdefgh\,*T}_{pq} \; \bar{i}^{abcdefgh}_{pq}$$

$$= [\, A_r \, \bar{v}_{pq}^{0,1,2,3,4,5,6,7}]^{*T} [\, A_r \, \bar{i}_{pq}^{0,1,2,3,4,5,6,7}]$$

$$= [\bar{v}_{pq}^{0,1,2,3,4,5,6,7}]^{*T} \; A_r^{*T} \; A_r \, [\, \bar{i}_{pq}^{0,1,2,3,4,5,6,7}]$$

$$= [\, \bar{v}_{pq}^{0,1,2,3,4,5,6,7}]^{*T} [\, \bar{i}_{pq}^{0,1,2,3,4,5,6,7} \,]$$

$$\text{as} \quad A_r^{*} = A_r \quad \text{and} \quad A_r^{*T} \, A_r = A_r^{T} \, A_r = I$$

Hence we conclude:

<u>Proposition</u>: The orthogonal matrix A_r which transforms the fiel of phasors of a 8-phase system to the field of components is a linear power invariant real transformation matrix similar to Clarke's component transformation matrix of 3-phase systems.

CHAPTER 4

12-PHASE POWER SYSTEM NETWORKS

In the previous chapter, we derived suitable transforma for the purpose of both steady state as well as transient anal of 8-phase systems. Here, we derive the transformations for t analysis of 12-phase systems and also the expression for seque impedances and complex power.

4.1 12-PHASE ROTATING ELEMENTS

For rotating elements, the symmetries are such that circularly permutations port voltages will cause similar perm tations of the port currents, i.e., if the voltage vector $\bar{v}_{pq}^{phase}$ is changed to $D(R)\ \bar{v}_{pq}^{phase}$ then correspondingly the cur vector $\bar{i}_{pq}^{phase}$ is replaced by $D(R)\ \bar{i}_{pq}^{phase}$. So

$$D(R)\ \bar{v}_{pq}^{phase} = [Z_{pq}]_{phase}\ D(R)\ \bar{i}_{pq}^{phase}$$

or

$$\bar{v}_{pq}^{phase} = D^{-1}(R)[Z_{pq}]_{phase}\ D(R)\ \bar{i}_{pq}^{phase} \qquad (4.$$

Comparing equation (2.4) and equation (4.1) we get

$$[Z_{pq}]_{phase} = D^{-1}(R)[Z_{pq}]_{phase}\ D(R)$$

Taking $R = C_{12}^{1}$, then $D^{-1}(C_{12}^{1}) = D(C_{12}^{11})$

$$[Z_{pq}]_{phase} = D(C_{12}^{11})\ [Z_{pq}]_{phase}\ D(C_{12}^{1}) \qquad (4$$

Performing the matrix multiplications and thenmaking term by t comparison in Eqn.(4.2), we find that $Z_{pq\ phase}$ is cyclic matr and is of the form: $Z_{pq\ phase} =$

$$\begin{bmatrix}
z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} \\
z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} \\
z_{pq}^{10} & z_{pq}^{11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} \\
z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} \\
z_{pq}^{8} & z_{pq}^{9} & z_{pq}^{10} & z_{pq}^{11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} \\
z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} \\
z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} \\
z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} \\
z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} \\
z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} \\
z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s} & z_{pq}^{m1} \\
z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{m3} & z_{pq}^{m4} & z_{pq}^{m5} & z_{pq}^{m6} & z_{pq}^{m7} & z_{pq}^{m8} & z_{pq}^{m9} & z_{pq}^{m10} & z_{pq}^{m11} & z_{pq}^{s}
\end{bmatrix}$$

Now, the eigenvectors of permutation matrices (eqnation 2.5) given by the following matrix A_c [11]

$$A_c = \frac{1}{\sqrt{12}}\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} \\
1 & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a \\
1 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 \\
1 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 \\
1 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 \\
1 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 \\
1 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 \\
1 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 \\
1 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 \\
1 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 \\
1 & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10}
\end{bmatrix}$$

Matrix A_c can also be expressed as

$$A_c = \frac{1}{\sqrt{12}}\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & a & a^2 & a^3 & -a^{*2} & -a^* & -1 & -a & -a^2 & -a^3 & a^{*2} & a^* \\
1 & a^2 & a^3 & -a^{*2} & -a^* & -1 & -a & -a^2 & -a^3 & a^{*2} & a^* & a \\
1 & a^3 & -a^{*2} & -a^* & -1 & -a & -a^2 & -a^3 & a^{*2} & a^* & a & a^2 \\
1 & -a^{*2} & -a^* & -1 & -a & -a^2 & -a^3 & a^{*2} & a^* & a & a^2 & a^3 \\
1 & -a^* & -1 & -a & -a^2 & -a^3 & a^{*2} & a^* & a & a^2 & a^3 & -a^{*2} \\
1 & -1 & -a & -a^2 & -a^3 & a^{*2} & a^* & a & a^2 & a^3 & -a^{*2} & -a^* \\
1 & -a & -a^2 & -a^3 & a^{*2} & a^* & a & a^2 & a^3 & -a^{*2} & -a^* & -1 \\
1 & -a^2 & -a^3 & a^{*2} & a^* & a & a^2 & a^3 & -a^{*2} & -a^* & -1 & -a \\
1 & -a^3 & a^{*2} & a^* & a & a^2 & a^3 & -a^{*2} & -a^* & -1 & -a & -a^2 \\
1 & a^{*2} & a^* & a & a^2 & a^3 & -a^{*2} & -a^* & -1 & -a & -a^2 & -a^3 \\
1 & a^* & a & a^2 & a^3 & -a^{*2} & -a^* & -1 & -a & -a^2 & -a^3 & a^{*2}
\end{bmatrix}$$

because $a = e^{j2\pi/12} = 1\angle 30° = \frac{1}{2}(\sqrt{3}+j1)$

$$a^2 = e^{j4\pi/12} = 1\angle 60° = \frac{1}{2}(1+j\sqrt{3})$$

$$a^3 = e^{j6\pi/12} = 1\angle 90° = j$$

$$a^4 = e^{j8\pi/12} = 1\angle 120° = \frac{1}{2}(-1+j\sqrt{3}) = -a^{*2}$$

$$a^5 = e^{j10\pi/12} = 1\angle 150° = \frac{1}{2}(-\sqrt{3}+j1) = -a^*$$

$$a^6 = e^{j12\pi/12} = 1\angle 180° = -1$$

$$a^7 = -a \; ; \quad a^8 = -a^2 \; ; \quad a^9 = a^{*3}; \quad a^{10} = a^{*2},$$

$$a^{11} = a^* \; ; \; a^{12} = 1$$

The unitary matrix A_c which diagonalizes the permutati matrix D(R), will also diagonalize the impedance matrix $\left[Z_{pq}\right]$ it commutes with D(R). The matrix A_c is the transformation ma for 12-phase power system network similar to symmetrical comp nent matrix for a 3-phase system.

Now, we rederive the transformation matrix A_c, using group theoretic techniques, for 12-phase system.

Group Theoretic Approach:

We have seen in Chapter 2 that permutation matrices (equation 2.5) representing rotational symmetries of a 12-ph power system.network form a cyclic group C_{12} of order 12. Ea element of group is in a separate class. Therefore, the num of classes in group C_{12} will be equal to 12 and the number c

irreducible representations which are equal to the number of classes will also be equal to 12. Let $l_1, l_2, \ldots, l_{12}$ be the dimensions of these irreducible representations, then from Eqn. (3.7), we get

$$\sum_{i=1}^{12} l_i^2 = l_1^2 + l_2^2 + \ldots + l_{12}^2 = 12$$

i.e. $\quad l_1 = l_2 = \ldots = l_{12} = 1$

Therefore all the irreducible representations have the dimensi equal to 1. Likewise from equation (3.8) $\sum_{i=1}^{12} X_i^2(R) = 12$ and h the characters $X^i(R)$ are each of unity modulus. Using standar computational techniques based on the orthogonality theorem (Appendix 2), one gets the character table shown below:

C_{12}	E	C_{12}^1	C_{12}^2	C_{12}^3	C_{12}^4	C_{12}^5	C_{12}^6	C_{12}^7	C_{12}^8	C_{12}^9	C_{12}^{10}	C_{12}^{11}
$X^1(R)$	1	1	1	1	1	1	1	1	1	1	1	1
$X^2(R)$	1	a	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}
$X^3(R)$	1	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	a
$X^4(R)$	1	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	a	a^2
$X^5(R)$	1	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	a	a^2	a^3
$X^6(R)$	1	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	a	a^2	a^3	a^4
$X^7(R)$	1	a^6	a^7	a^8	a^9	a^{10}	a^{11}	a	a^2	a^3	a^4	a^5
$X^8(R)$	1	a^7	a^8	a^9	a^{10}	a^{11}	a	a^2	a^3	a^4	a^5	a^6
$X^9(R)$	1	a^8	a^9	a^{10}	a^{11}	a	a^2	a^3	a^4	a^5	a^6	a^7
$X^{10}(R)$	1	a^9	a^{10}	a^{11}	a	a^2	a^3	a^4	a^5	a^6	a^7	a^8
$X^{11}(R)$	1	a^{10}	a^{11}	a	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9
$X^{12}(R)$	1	a^{11}	a	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}

(4

Evidently, in this case as the dimension of irreducible represen- tations is 1, the irreducible representation is given by the character table itself.

C_{12}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{1}^{1}
$D^{1}(R)$	1	1	1	1	1	1	1	1	1	1	1	1
$D^{2}(R)$	1	a	a^{2}	a^{3}	a^{4}	a^{5}	a^{6}	a^{7}	a^{8}	a^{9}	a^{10}	a^{1}
$D^{3}(R)$	1	a^{2}	a^{3}	a^{4}	a^{5}	a^{6}	a^{7}	a^{8}	a^{9}	a^{10}	a^{11}	a
$D^{4}(R)$	1	a^{3}	a^{4}	a^{5}	a^{6}	a^{7}	a^{8}	a^{9}	a^{10}	a^{11}	a	a
$D^{5}(R)$	1	a^{4}	a^{5}	a^{6}	a^{7}	a^{8}	a^{9}	a^{10}	a^{11}	a	a^{2}	a
$D^{6}(R)$	1	a^{5}	a^{6}	a^{7}	a^{8}	a^{9}	a^{10}	a^{11}	a	a^{2}	a^{3}	a
$D^{7}(R)$	1	a^{6}	a^{7}	a^{8}	a^{9}	a^{10}	a^{11}	a	a^{2}	a^{3}	a^{4}	a
$D^{8}(R)$	1	a^{7}	a^{8}	a^{9}	a^{10}	a^{11}	a	a^{2}	a^{3}	a^{4}	a^{5}	a
$D^{9}(R)$	1	a^{8}	a^{9}	a^{10}	a^{11}	a	a^{2}	a^{3}	a^{4}	a^{5}	a^{6}	a
$D^{10}(R)$	1	a^{9}	a^{10}	a^{11}	a	a^{2}	a^{3}	a^{4}	a^{5}	a^{6}	a^{7}	a
$D^{11}(R)$	1	a^{10}	a^{11}	a	a^{2}	a^{3}	a^{4}	a^{5}	a^{6}	a^{7}	a^{8}	a
$D^{12}(R)$	1	a^{11}	a	a^{2}	a^{3}	a^{4}	a^{5}	a^{6}	a^{7}	a^{8}	a^{9}	a

Irreducible Representation (4

The number of times each of these irreducible representations $D^{i}(R)$ appear at the diagonal of the reduced out representatio $\bar{D}(R)$ is given by Eqn.(3.10),

$$a_i = \frac{1}{h} \sum_R X^i(R)\ X(R)$$

Substituting from Eqn.(4.5), we get

$$a_1 = \frac{1}{12} \sum_R X^1(R)X(R) = \frac{1}{12}(1(12)+1(0)+1(0)+1(0)+1(0)+1(0)+1(\ + 1(0)+1(0)+1(0)+1(0)+1(0)) = 1$$

Similarly, $a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = a_9 = a_{10} = a_{11}$

$= a_{12} = 1$

Therefore $\bar{D}(R)$ is of the form:

$$\bar{D}(R) = \begin{bmatrix} D^1(R) & & & & & & & & & & & \\ & D^2(R) & & & & & & & 0 & & & \\ & & D^3(R) & & & & & & & & & \\ & & & D^4(R) & & & & & & & & \\ & & & & D^5(R) & & & & & & & \\ & & & & & D^6(R) & & & & & & \\ & & & & & & D^7(R) & & & & & \\ & & & & & & & D^8(R) & & & & \\ & & & & & & & & D^9(R) & & & \\ & & & & & & & & & D^{10}(R) & & \\ & & 0 & & & & & & & & D^{11}(R) & \\ & & & & & & & & & & & D^{12}(R) \end{bmatrix} \qquad (4.7$$

Now, we construct the matrices G_i^j using Eqn.(3.16), i.e.

$$G_i^j = \sum_R D^j(R)_{ii} \; D(R) \qquad \text{for } j = 1,2,\ldots, 12$$

$$G_1^1 = \sum_R D^1(R)_{11} D(R) = 1D(E)+1D(R_1)+1D(R_2)+1D(R_3)+1D(R_4)+1D(R_5) +1D(R_6)+1D(R_7)+1D(R_8)+ 1D(R_9)+1D(R_{10})+1D(R$$

where $R_i = C_{12}^i$ for $i = 1,2,\ldots, 11$.

$$G_1^2 = \sum_R D^2(R)_{11} D(R) = 1D(E)+aD(R_1)+a^2D(R_2)+a^3D(R_3)+a^4D(R_4)+a^5D +a^6D(R_6)+a^7D(R_7)+a^8D(R_8)+a^9D(R_9)+a^{10}D(R_1 +a^{11}D(R_{11})$$

$$G_1^3 = \sum_R D^3(R)_{11}D(R) = 1D(E)+a^2D(R_1)+a^3D(R_2)+a^4D(R_3)+a^5D(R_4)+a^6D(R_5)$$
$$+a^7D(R_6)+a^8D(R_7)+a^9D(R_8)+a^{10}D(R_9)+a^{11}D(R_{10})$$
$$+aD(R_{11}),$$

$$G_1^4 = \sum_R D^4(R)_{11}D(R) = 1D(E)+a^3D(R_1)+a^4D(R_2)+a^5D(R_3)+a^6D(R_4)+a^7D(R_5)$$
$$+a^8D(R_6)+a^9D(R_7)+a^{10}D(R_8)+a^{11}D(R_9)+aD(R_{10})$$
$$+a^2D(R_{11}),$$

$$G_1^5 = \sum_R D^5(R)_{11}D(R) = 1D(E)+a^4D(R_1)+a^5D(R_2)+a^6D(R_3)+a^7D(R_4)+a^8D(R_5)$$
$$+a^9D(R_6)+a^{10}D(R_7)+a^{11}D(R_8)+aD(R_9)+a^2D(R_{10})$$
$$+a^3D(R_{11}),$$

$$G_1^6 = \sum_R D^6(R)_{11}D(R) = 1D(E)+a^5D(R_1)+a^6D(R_2)+a^7D(R_3)+a^8D(R_4)+a^9D(R_5)$$
$$+a^{10}D(R_6)+a^{11}D(R_7)+aD(R_8)+a^2D(R_9)+a^3D(R_{10})$$
$$+a^4D(R_{11}),$$

$$G_1^7 = \sum_R D^7(R)_{11}D(R) = 1D(E)+a^6D(R_1)+a^7D(R_2)+a^8D(R_3)+a^9D(R_4)+a^{10}D(R_5)$$
$$+a^{11}D(R_6)+aD(R_7)+a\ D(R_8)+a^3D(R_9)+a^4D(R_{10})$$
$$+a^5D(R_{11}),$$

$$G_1^8 = \sum_R D^8(R)_{11}D(R) = 1D(E)+a^7D(R_1)+a^8D(R_2)+a^9D(R_3)+a^{10}D(R_4)$$
$$+a^{11}D(R_5)+aD(R_6)+a^2D(R_7)+a^3D(R_8)+a^4D(R_9)$$
$$+a^5D(R_{10})+a^6D(R_{11}),$$

$$G_1^9 = \sum_R D^9(R)_{11}D(R) = 1D(E)+a^8D(R_1)+a^9D(R_2)+a^{10}D(R_3)+a^{11}D(R_4)$$
$$+aD(R_5)+a^2D(R_6)+a^3D(R_7)+a^4D(R_8)+a^5D(R_9)$$
$$+a^6D(R_{10})+a^7D(R_{11}),$$

$$G_1^{10} = \sum_R D^{10}(R)_{11}D(R) = 1D(E)+a^9D(R_1)+a^{10}D(R_2)+a^{11}D(R_3)+aD(R_4)$$
$$+a^2D(R_5)+a^3D(R_6)+a_4D(R_7)+a^5D(R_8)+a_6D(R_9)$$
$$+a^7D(R_{10})+a^8D(R_{11}),$$

$$G_1^{11} = \sum_R D^{11}(R)_{11}D(R) = 1D(E)+a^{10}D(R_1)+a^{11}D(R_2)+aD(R_3)+a^2D(R_4)$$
$$+a^3D(R_5)+a^4D(R_6)+a^5D(R_7)+a^6D(R_8)+a^7D(R_9)$$
$$+a^8D(R_{10})+a^9D(R_{11}),$$

and

$$G_1^{12} = \sum_R D^{12}(R)_{11}D(R) = 1D(E)+a^{11}D(R_1)+aD(R_2)+a^2D(R_3)+a^3\ D(R_4)$$
$$+a^4D(R_5)+a^5D(R_6)+a^6D(R_7)+a^7D(R_8)+a^8D(R_9)$$
$$+a^9D(R_{10})+a^{10}D(R_{11}).$$

Now, scanning the matrices G_i^j's from left to right and picking first linearly independent columns from each of these matrices we get the basis vector α_{mnp}. The basis vectors after normalization to unity come out to be

$$\alpha_{111} = \frac{1}{\sqrt{12}}\,[\,1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\,]^T$$

$$\alpha_{211} = \frac{1}{\sqrt{12}}\,[\,1\quad a\quad a^2\quad a^3\quad a^4\quad a^5\quad a^6\quad a^7\quad a^8\quad a^9\quad a^{10}\quad a^{11}\,]^T$$

$$\alpha_{311} = \frac{1}{\sqrt{12}}\,[\,1\quad a^2\quad a^3\quad a^4\quad a^5\quad a^6\quad a^7\quad a^8\quad a^9\quad a^{10}\quad a^{11}\quad a\,]^T$$

$$\alpha_{411} = \frac{1}{\sqrt{12}}\,[\,1\quad a^3\quad a^4\quad a^5\quad a^6\quad a^7\quad a^8\quad a^9\quad a^{10}\quad a^{11}\quad a\quad a^2\,]^T$$

$$\alpha_{511} = \frac{1}{\sqrt{12}}\,[\,1\quad a^4\quad a^5\quad a^6\quad a^7\quad a^8\quad a^9\quad a^{10}\quad a^{11}\quad a\quad a^2\quad a^3\,]^T$$

$$\alpha_{611} = \frac{1}{\sqrt{12}} [1 \quad a^5 \; a^6 \; a^7 \; a^8 \; a^9 \; a^{10} \; a^{11} \; a \quad a^2 \; a^3 \; a^4]^T$$

$$\alpha_{711} = \frac{1}{\sqrt{12}} [1 \quad a^6 \; a^7 \; a^8 \; a^9 \; a^{10} \; a^{11} \; a \quad a^2 \; a^3 \; a^4 \; a^5]^T$$

$$\alpha_{811} = \frac{1}{\sqrt{12}} [1 \quad a^7 \; a^8 \; a^9 \; a^{10} \; a^{11} \; a \quad a^2 \; a^3 \; a^4 \; a^5 \; a^6]^T$$

$$\alpha_{911} = \frac{1}{\sqrt{12}} [1 \quad a^8 \; a^9 \; a^{10} \; a^{11} \; a \quad a^2 \; a^3 \; a^4 \; a^5 \; a^6 \; a^7]^T$$

$$\alpha_{10\,11} = \frac{1}{\sqrt{12}} [1 \quad a^9 \; a^{10} \; a^{11} \; a \quad a^2 \; a^3 \; a^4 \; a^5 \; a^6 \; a^7 \; a^8]^T$$

$$\alpha_{11\,11} = \frac{1}{\sqrt{12}} [1 \quad a^{10} \; a^{11} \; a \quad a^2 \; a^3 \; a^4 \; a^5 \; a^6 \; a^7 \; a^8 \; a^9]^T$$

$$\alpha_{12\,11} = \frac{1}{\sqrt{12}} [1 \quad a^{11} \; a \quad a^2 \; a^3 \; a^4 \; a^5 \; a^6 \; a^7 \; a^8 \; a^9 \; a^{10}]^T$$

Thus the transformation matrix $A_c = \alpha$ comes out to be

$$A_c = \begin{array}{cccccccccccc} \alpha_{111} & \alpha_{211} & \alpha_{311} & \alpha_{411} & \alpha_{511} & \alpha_{611} & \alpha_{711} & \alpha_{811} & \alpha_{911} & \alpha_{10\,11} & \alpha_{11\,11} & \alpha_{12\,11} \\ D^1(R) & D^2(R) & D^3(R) & D^4(R) & D^5(R) & D^6(R) & D^7(R) & D^8(R) & D^9(R) & D^{10}(R) & D^{11}(R) & D^{12}(R) \end{array}$$

$$A_c = \frac{1}{\sqrt{12}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} \\ 1 & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a \\ 1 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 \\ 1 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 \\ 1 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 \\ 1 & a^6 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 \\ 1 & a^7 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 \\ 1 & a^8 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 \\ 1 & a^9 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 \\ 1 & a^{10} & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 \\ 1 & a^{11} & a & a^2 & a^3 & a^4 & a^5 & a^6 & a^7 & a^8 & a^9 & a^{10} \end{bmatrix} \quad (4.8$$

We can see that this matrix is same as the one given in Eqn.(4

It can be seen that $A_c^{*T} A_c = I$ i.e. A_c is unitary and power invariant. Now from Eqn.(3.17),

$$[Z_{pq}]_{comp} = A_c^{*T} [Z_{pq}]_{phase} A_c$$

Substituting for A_c from eqn.(4.4) and $[Z_{pq}]_{phase}$ from (4.3), we get

$$Z_{pq\ comp}^{0,1,2,3,4,5,6,7,8,9,10,11} =$$

$$\begin{bmatrix} z_{pq}^{s}+z_{pq}^{m1}+z_{pq}^{m2}+z_{pq}^{m3}+z_{pq}^{m4}+z_{pq}^{m5}+z_{pq}^{m6}+z_{pq}^{m7}+z_{pq}^{m8}+z_{pq}^{m9}+z_{pq}^{m10}+z_{pq}^{m11} & & 0 \\ & z_{pq}^{s}+az_{pq}^{m1}+a^2z_{pq}^{m2}+a^3z_{pq}^{m3}-a^{*2}z_{pq}^{m4}-a^{*}z_{pq}^{m5}-z_{pq}^{m6}-az_{pq}^{m7}-a^2z_{pq}^{m8} & \\ & a^3z_{pq}^{m9}+a^{*2}z_{pq}^{m10}+a^{*}z_{pq}^{m11} & \\ & \cdot & \\ 0 & \cdot & \\ & \cdot & \end{bmatrix}$$

Therefore, the zero sequence impedance z_{pq}^{o} is equal to

$$z_{pq}^{o} = z_{pq}^{s} + z_{pq}^{m1} + z_{pq}^{m2} + z_{pq}^{m3} + z_{pq}^{m4} + z_{pq}^{m5} + z_{pq}^{m6} + z_{pq}^{m7} + z_{pq}^{m8} + z_{pq}^{m9} + z_{pq}^{m10} + z_{pq}^{m11}$$

The first sequence impedance

$$z_{pq}^{1} = z_{pq}^{s} + az_{pq}^{m1} + a^{2}z_{pq}^{m2} + a^{3}z_{pq}^{m3} - a^{*2}z_{pq}^{m4} - a^{*}z_{pq}^{m5} - z_{pq}^{m6} - az_{pq}^{m7} - a^{2}z_{pq}^{m8} - a^{3}z_{pq}^{m9} + a^{*2}z_{pq}^{m10} + a^{*}z_{pq}^{m11}$$

and so on.

Proposition: The transformation matrix A_c which diagonalizes the coefficient matrix of 12-phase rotating element network, i a linear power invariant transformation matrix with complex elements similar to the symmetrical components for 3-phase system.

4.2 12-PHASE STATIONARY ELEMENTS

Power system stationary elements possess rotational as well as reflection symmetries (typical example is that of 12- transposed transmission line). These symmetries can be repre ted by permutation matrices D(R) (eqns. 2.5 and 2.7) as seen last chapter. In the previous section we have seen that impedance matrix for 12-phase network with rotational symmet is cyclic (eqn.4.3). Now, for networks possessing reflectio

symmetries in addit on to rotational ones, applying (3.1) for $R = \delta'_a, \delta'_b, \delta'_c, \delta'_d, \delta'_e, \delta'_f, \delta'_g, \delta'_h, \delta'_i, \delta'_j, \delta'_k, \delta'_l$ and making term by term comparison in

$$[Z_{pq}]^{phase} = D^{-1}(R)[Z_{pq}]_{phase}\, D(R)$$

$$\text{for } R = \delta'_a, \delta'_b, \ldots, \delta'_l$$

We get that $[Z_{pq}]$ is of the form:

$[Z_{pq}]_{phase} =$

$$\begin{bmatrix}
z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq} & z^m_{pq} \\
z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^m_{pq} & z^s_{pq}
\end{bmatrix}$$

(4.10)

The twenty four symmetry operations form the group C_{12v} of ord twenty four but there are only nine classes viz. $[D(E)]$; $[D(C^1_{12})$, $D(C^{11}_{12})]$; $[D(C^2_{12}), D(C^{10}_{12})]$; $[D(C^3_{12}), D(C^9_{12})]$; $[D(C^4_{12}), D(C^8_{12})]$ $[D(C^5_{12}), D(C^7_{12})]$; $[D(C^6_{12})]$; $[D(\delta'_a), D(\delta'_b), D(\delta'_c), D(\delta'_d), D(\delta'_e)$ $D(\delta'_f)]$; $[D(\delta'_g), D(\delta'_h), D(\delta'_i), D(\delta'_j), D(\delta'_k), D(\delta'_l)]$. Therefor the number of irreducible representation is also nine. Let $l_1, l_2, \ldots, l_9$ be the dimensions of these irreducible represe tations. Then, from eqn.(3.7), we have

$$\sum_{i=1}^{9} l_i^2 = l_1^2 + l_2^2 + \ldots. + l_9^2 = h = 24$$

The solution of this equation is $l_1 = l_2 = l_3 = l_4 = 1$ and $l_5 = l_6 = l_7 = l_8 = l_9 = 2$. From this we conclude that there are four irreducible representations viz. $D^1(R)$, $D^2(R)$, $D^3(R)$ $D^4(R)$ of dimension 1 each and five irreducible representation viz. $D^5(R)$, $D^6(R)$, $D^7(R)$, $D^8(R)$,and $D^9(R)$ of dimension 2 each From eqn.(3.8),

$$\sum_R X_i^2(R) = h = 24$$

So, the character of each of twenty four of first four irredu representations whose dimension is 1 is of unity modulus, but of the last five representations whose dimension is 2, is 2, -2 or 0. Now, we write character table using orthogonality theorem (Appendix 2) and its consequences.

C_{12v}	E	C_{12}^1	C_{12}^2	C_{12}^3	C_{12}^4	C_{12}^5	C_{12}^6	C_{12}^7	C_{12}^8	C_{12}^9	C_{12}^{10}	C_{12}^{11}	δ'_a	δ'_b	δ'_c	δ'_d	δ'_e	δ'_f	δ'_g	δ'_h	δ'_i	δ'_j	δ'_k	δ'_l
$X^1(R)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X^2(R)$	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
$X^3(R)$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
$X^4(R)$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
$X^5(R)$	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
$X^6(R)$	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
$X^7(R)$	2	0	-2	0	2	0	-2	0	2	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
$X^8(R)$	2	1	1	-2	-1	-1	-2	-1	-1	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
$X^9(R)$	2	-1	1	2	-1	1	-2	1	-1	-2	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
Red.Rep $X(R)$	12	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	0	0	0	0	0	0

CHARACTER TABLE (4.11)

It is evident that irreducible representations whose dimension is 1, are the same as their characters. Irreducible representations, whose dimension is 2 are found by using orthogonality theorem and its consequences (Appendix 2) and Cayley's table. To find the irreducible representations, first we find irreducible representation for rotational symmetries and by using Cayley's table irreducible representations for reflection symmetries are found. The irreducible representations in the final form are given by equation (4.12).

The number of times each of the irreducible representation $D^i(R)$ appears at the diagonal of $\overline{D}(R)$ is determined by eqn.(3.10), and thus we obtain,

$$a_1 = 1,\ a_2 = 0,\ a_3 = 1,\ a_4 = 0,\ a_5 = a_6 = a_7 = a_8 = a_9 = 1$$

Let the transformation matrix be A_r, then

$$\overline{D}(R) = A_r^{*T} D(R) A_r = \begin{bmatrix} D^1(R) & & & & & & 0 \\ & D^3(R) & & & & & \\ & & D^5(R) & & & & \\ & & & D^6(R) & & & \\ & & & & D^7(R) & & \\ & & & & & D^8(R) & \\ 0 & & & & & & D^9(R) \end{bmatrix} \tag{4.13}$$

After this we determine matrix G_i^j following the procedure outlined earlier and thus obtain G_1^1, G_2^1, G_3^1, G_4^1, G_5^1, G_5^2, G_6^1, G_6^2, G_7^1, G_7^2, G_8^1, G_8^2, G_9^1, and G_9^2.

C_{12v}	E	C_{12}^{1}	C_{12}^{2}	C_{12}^{3}	C_{12}^{4}	C_{12}^{5}	C_{12}^{6}
$D^1[R]$	1	1	1	1	1	1	1
$D^2[R]$	1	1	1	1	1	1	1
$D^3[R]$	1	-1	1	-1	1	-1	1
$D^4[R]$	1	-1	1	-1	1	-1	1
$D^5[R]$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
$D^6[R]$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
$D^7[R]$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	$\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$
$D^8[R]$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$
$D^9(R)$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

C_{12v}	C_{12}^{7}	C_{12}^{8}	C_{12}^{9}	C_{12}^{10}	C_{12}^{11}
$D^1[R]$	1	1	1	1	1
$D^2[R]$	1	1	1	1	1
$D^3[R]$	-1	1	-1	1	-1
$D^4[R]$	-1	1	-1	1	-1
$D^5[R]$	$\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$
$D^6[R]$	$\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$
$D^7[R]$	$\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	$\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$
$D^8[R]$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$
$D^9(R)$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$

Contd.....

$$D^4(R) \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1$$

$$D^3(R) \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1$$

$$D^3(R) \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$D^5(R) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$$

$$D^6(R) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$D^7(R) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$D^8(R) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$$

$$D^9(R) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \tag{4.12}$$

The basis vector α_{mnp} is also found in the similar manner and comes out to be the following:

$$\alpha_{111} = \frac{1}{\sqrt{12}} [\; 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \;]^T$$

$$\alpha_{311} = \frac{1}{\sqrt{12}} [\; 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \;]^T$$

$$\alpha_{511} = \frac{1}{\sqrt{12}} [\; \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \quad -\sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \quad -\sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \;]^T$$

$$\alpha_{521} = \frac{1}{\sqrt{12}} [\; 0 \quad \sqrt{\frac{3}{2}} \quad \sqrt{\frac{3}{2}} \quad 0 \quad -\sqrt{\frac{3}{2}} \quad -\sqrt{\frac{3}{2}} \quad 0 \quad \sqrt{\frac{3}{2}} \quad \sqrt{\frac{3}{2}} \quad 0 \quad -\sqrt{\frac{3}{2}} \quad -\sqrt{\frac{3}{2}} \;]^T$$

$$\alpha_{611} = \frac{1}{\sqrt{12}} [\; \sqrt{2} \quad -\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}} \quad \sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \quad \sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \quad \sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \;]^T$$

$$\alpha_{621} = \frac{1}{\sqrt{12}} [\; 0 \quad \sqrt{\frac{3}{2}} \quad -\sqrt{\frac{3}{2}} \quad 0 \quad \sqrt{\frac{3}{2}} \quad -\sqrt{\frac{3}{2}} \quad 0 \quad \sqrt{\frac{3}{2}} \quad -\sqrt{\frac{3}{2}} \quad 0 \quad \sqrt{\frac{3}{2}} \quad -\sqrt{\frac{3}{2}} \;]^T$$

$$\alpha_{711} = \frac{1}{\sqrt{12}} [\; \sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0 \;]^T$$

$$\alpha_{721} = \frac{1}{\sqrt{12}} [\; 0 \quad \sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 0 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 0 \quad -\sqrt{2} \;]^T$$

$$\alpha_{811} = \frac{1}{\sqrt{12}} [\; \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \sqrt{\frac{1}{2}} \quad -\sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \quad -\sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \quad \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \;]^T$$

$$\alpha_{821} = \frac{1}{\sqrt{12}} [\; 0 \quad \frac{1}{2}\sqrt{\frac{3}{2}} \quad \frac{3}{2}\sqrt{\frac{3}{2}} \quad \sqrt{\frac{3}{2}} \quad \frac{-1}{2}\sqrt{\frac{3}{2}} \quad \frac{1}{2}\sqrt{\frac{3}{2}} \quad 0 \quad \frac{1}{2}\sqrt{\frac{3}{2}} \quad -\frac{3}{2}\sqrt{\frac{3}{2}} \quad -\sqrt{\frac{3}{2}} \quad \frac{1}{2}\sqrt{\frac{3}{2}}2 \quad -\frac{1}{2}\sqrt{\frac{3}{2}} \;]$$

$$\alpha_{911} = \frac{1}{\sqrt{12}} [\; \sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad \sqrt{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad -\sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \quad -\sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}} \;]^T$$

and

$$\alpha_{921} = \frac{1}{\sqrt{12}}\left[0 \;\; \tfrac{1}{2}\sqrt{\tfrac{3}{2}} \;\; -\tfrac{3}{2}\sqrt{\tfrac{3}{2}} \;\; \sqrt{\tfrac{3}{2}} \;\; \tfrac{1}{2}\sqrt{\tfrac{3}{2}} \;\; \tfrac{1}{2}\sqrt{\tfrac{3}{2}} \;\; 0 \;\; -\tfrac{1}{2}\sqrt{\tfrac{3}{2}} \;\; \tfrac{3}{2}\sqrt{\tfrac{3}{2}} \;\; -\sqrt{\tfrac{3}{2}} \;\; -\tfrac{1}{2}\sqrt{\tfrac{3}{2}} \;\; -\tfrac{1}{2}\sqrt{\tfrac{3}{2}}\right]^T$$

Therefore, transformation matrix A_r = matrix α =

$$[\alpha_{111} \;\; \alpha_{311} \;\; \alpha_{511} \;\; \alpha_{521} \;\; \alpha_{611} \;\; \alpha_{621} \;\; \alpha_{711} \;\; \alpha_{721} \;\; \alpha_{811} \;\; \alpha_{821} \;\; \alpha_{911} \;\; \alpha_{921}]$$

$D^1(R)$ $D^3(R)$ $D^5(R)$ $D^6(R)$ $D^7(R)$ $D^8(R)$ $D^9(R)$

$$= \frac{1}{\sqrt{12}}\begin{bmatrix}
1 & 1 & \sqrt{2} & 0 & \sqrt{2} & 0 & \sqrt{2} & 0 & \sqrt{2} & 0 & \sqrt{2} & 0 \\
1 & -1 & \frac{1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & 0 & \sqrt{2} & \frac{1}{\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} \\
1 & 1 & \frac{-1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & -\sqrt{2} & 0 & \frac{1}{\sqrt{2}} & \frac{3}{2}\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} & -\frac{3}{2}\sqrt{\frac{3}{2}} \\
1 & -1 & -\sqrt{2} & 0 & \sqrt{2} & 0 & 0 & -\sqrt{2} & -\sqrt{2} & \sqrt{\frac{3}{2}} & \sqrt{2} & \sqrt{\frac{3}{2}} \\
1 & 1 & \frac{-1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & \sqrt{2} & 0 & \frac{-1}{\sqrt{2}} & -\frac{1}{2}\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} \\
1 & -1 & \frac{1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & -\frac{3}{2} & 0 & -\sqrt{2} & \frac{-1}{\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} \\
1 & 1 & \sqrt{2} & 0 & \sqrt{2} & 0 & -\sqrt{2} & 0 & -\sqrt{2} & 0 & -\sqrt{2} & 0 \\
1 & -1 & \frac{1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & 0 & \sqrt{2} & \frac{-1}{\sqrt{2}} & -\frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{2}\sqrt{\frac{3}{2}} \\
1 & 1 & \frac{-1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & \sqrt{2} & 0 & \frac{-1}{\sqrt{2}} & -\frac{3}{2}\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & \frac{3}{2}\sqrt{\frac{3}{2}} \\
1 & -1 & -\sqrt{2} & 0 & \sqrt{2} & 0 & 0 & -\sqrt{2} & \sqrt{2} & -\sqrt{\frac{3}{2}} & -\sqrt{2} & -\sqrt{\frac{3}{2}} \\
1 & 1 & \frac{-1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & -\sqrt{2} & 0 & \frac{+1}{\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{2}\sqrt{\frac{3}{2}} \\
1 & -1 & \frac{1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & 0 & \sqrt{2} & \frac{+1}{\sqrt{2}} & -\frac{1}{2}\sqrt{\frac{3}{2}} & \frac{-1}{\sqrt{2}} & -\frac{1}{2}\sqrt{\frac{3}{2}}
\end{bmatrix} \qquad (4.14)$$

It can be verified that $A_r^T A_r = I = A_r^{-1} A_r$. Therefore, the transformation matrix A_r is orthogonal. From equation (3.17),

$$[Z_{pq}]_{comp} = A_r^{*T} [Z_{pq}]_{phase} A_r$$

Substituting for $[Z_{pq}]_{phase}$ from equation (4.10) and A_r from (4.14) and then performing the matrix multiplications, we get

$$[Z_{pq}]_{comp}^{0,1,2,3,4,5,6,7,8,9,10,11} =$$

$$= \begin{bmatrix}
z^s_{pq}+11z^m_{pq} & & & & & & & & & & & \\
& z^s_{pq}-z^m_{pq} & & & & & & 0 & & & & \\
& & z^s_{pq}-z^m_{pq} & & & & & & & & & \\
& & & z^s_{pq}-z^m_{pq} & & & & & & & & \\
& & & & z^s_{pq}-z^m_{pq} & & & & & & & \\
& & & & & z^s_{pq}-z^m_{pq} & & & & & & \\
& & & & & & z^s_{pq}-z^m_{pq} & & & & & \\
& & & & & & & z^s_{pq}-z^m_{pq} & & & & \\
& & & & & & & & z^s_{pq}-z^m_{pq} & & & \\
& & & & & & & & & z^s_{pq}-z^m_{pq} & & \\
& & & 0 & & & & & & & z^s_{pq}-z^m_{pq} & \\
& & & & & & & & & & & z^s_{pq}-z^m_{pq}
\end{bmatrix} \qquad (4.15)$$

From this, it is clear that the transformation matrix A_r diagonalizes the coefficient matrix $[Z_{pq}]$ of 12-phase stationary elements.

It is to be noted here that the matrix A_c derived earlier can also diagonalize $[Z_{pq}]$ since the stationary elements also possess rotational symmetries in addition to reflection ones. The diagonal elements of $[Z_{pq}]_{comp}$ are the sequence impedances in this case. More specifically, the zero sequence impedance

$$z^o_{pq} = z^s_{pq} + 11\ z^m_{pq}$$

First sequence impedance $z^1_{pq} = z^s_{pq} - z^m_{pq}$

The second to eleventh sequence impedances are same as z^1_{pq} i.e. first sequence impedance.

<u>Complex Power</u>

The complex power in the 12-phase stationary element p-q

$$S_{pq} = P_{pq} - jQ_{pq} = [\ \bar{v}^{abcdefghijkl}_{pq}\ ]^{*T} [\ \bar{i}^{abcdefghijkl}_{pq}\ ]$$

$$= [\ A_r \cdot \bar{v}^{0,1,2,3,4,5,6,7,8,9,10,11}_{pq}\ ]^{*T} [\ A_r \bar{i}^{0,1,2,3,4,5,6,7,8,9,10,11}_{pq}\ ]$$

$$= [\ \bar{v}^{0,1,2,3,4,5,6,7,8,9,10,11}_{pq}\ ]^{*T}\ A_r^{*T} A_r \times [\ \bar{i}^{0,1,2,3,4,5,6,7,8,9,10,11}_{pq}\ ]$$

$$= [\ \bar{v}^{0,1,2,3,4,5,6,7,8,9,10,11}_{pq}\ ]^{*T} \times [\ i^{0,1,2,3,4,5,6,7,8,9,10,11}_{pq}\ ] \qquad (4.16)$$

because $A^*_r = A_r$ and $A^{*T}_r A_r = A^T_r A_r = I$ i.e. $A^{-1}_r = A^T_r$.

Thus we conclude,

Proposition: The orthogonal matrix A_r which transforms the field of phasors of a 12-phase system to the field of components is a linear power invariant real transformation matrix similar to Clarke's component transformation matrix of 3-phase systems.

Remark: Similar expression for complex power is also obtained using complex transformation matrix A_c.

CHAPTER 5

CONCLUSION

The inherent symmetries associated with the power system networks enable us to simplify the analysis of multiphase power system networks by application of group theoretic techniques. An attempt has been made in this thesis to make use of this fact in the steady state analysis of multiphase power system networks.

The methods developed earlier, for 4-phase and 6-phase power systems have been used here for analysis of 8-phase and 12-phase systems. Based on the fact that symmetries of n-phase power system network with rotational elements constitute a cyclic group C_n and that of networks with stationary elements constitute a group C_{nv}, similarity transformations with complex elements as well as real elements for 8 phase and 12 phase power system networks have been developed using group theoretic techniques. The main advantage of using group theoretic techniques is that they are applicable in a unified manner to multiphase system which seem to have a bright future. The similarity transformations developed here are linear and power invariant.

Amongst the new directions in which these techniques may be applied is one of transient analysis of multiphase systems by using the symmetries of network under transient conditions. Another direction in which the techniques

used in this thesis may have a possible application is in the analysis of network with nonlinear elements such as saturable core reactors. These elements although nonlinear, do exhibit symmetries analogus to those of the linear elements. With the help of approximate matrix representation of these symmetries, the analysis of networks containing nonlinear elements is likely to be considerably simplified.

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APPENDIX 1

We give here a brief review of the mathematical theory of groups used in this thesis. We start first with the definition and properties of the finite group. For details, reader are required to refer to the references [1,16].

1. GROUP: Suppose we are given a set G. Let '.' (commonly known as multiplication) be the binary operation defined on the set G. Then set G is said to be a group if it satisfies the following postulates, called group axioms:

(i) Closure: Given any two elements a and b of the set G, a.b the result of the binary operation on a and b is also in G.

(ii) Associativity: For elements a,b and c of set G, we have the following relation,

$$a.(b.c) = (a.b).c$$

(iii) Existence of Identity: Among the elements of the set G, there exists an element e called identity element such that

$$a.e. = e.a = a$$

and (iv) Existence of Inverse: Corresponding to every element a of the set G, there exists an element a^{-1}, called the inverse of the element a of the set G such that

$$a.a^{-1} = a^{-1} a = e$$

If, in addition to the above four group axioms, the following condition is also satisfied, then the group is known as <u>commutative or abelian group</u>.

(v) Commutativity: For any element a and b in the group G, we have $a.b = b.a$

If the number of elements is finite in the group, then the group is said to be finite and then the number of distinct elements in the finite group is called the order of the group.

Group Multiplication Table:

For a finite group G with binary operation of multiplication, the multiplications of group elements and their products can most conveniently be presented in a table known as a group multiplication table. The group elements are arranged along the column and the row of the table. The entry in the i-jth position of the table is the group element $p_i.p_j$ which results from multiplication of p_i, an element in the ith row and p_j, an element in the jth row. For a completely and uniquely defined group, its elements which may have physical interpretation, can be represented by abstract quantities viz. a,b,c,.... etc.

Cyclic Group:

A cyclic group is one in which all elements are generated by a single elements, known as a generating elements or simply a generator. For example, the set G with elements as 4th root of unity $(1, a, a^2, a^3)$ where $a = e^{j2\pi/4}$ constitute a cyclic group under multiplication and element a is the generator element.

Classes:

Two elements a and b of a group G are conjugate if there exists another element X in G such that

$$b = X^{-1}aX$$

Conjugate elements have following properties:

(i) Every element is conjugate with itself.

(ii) If the element a is conjugate to b, then the element b will also be conjugate to a.

and

(iii) If a is conjugate to both b and c, then b and c are conjugate to one another.

A complete set of elements of a group G which are conjugate to one another is called a class of the group.

Note: The orders of all classes in a group, are integral factors of the order of the group.

APPENDIX 2

ORTHOGONALITY THEOREM

Several important properties of group representations and their characters are derived from basic theorem concerning elements of matrices which constitute irreducible representations of the group. This theorem is known as orthogonality theorem and is stated as follows:

$$\sum_R D^i(R)_{mn} \; D^j(R)_{m'n'}{}^* = \frac{h^2}{\sqrt{l_i l_j}} \delta_{ij} \; \delta_{mm'} \; \delta_{nn'} \qquad \text{(A-2.1)}$$

where h is the orderof the group and l_i is the dimension of the ith irreducible representation which is the order of each of the matrices constituting ith representation, $D^i(R)_{mn}$ is the element in the mth row and nth column of $D^i(R)$, the matrix corresponding to symmetry operation R in the ith irreducible representation. Eqn.(A2.1) of the orthogonality theorem can be split up into three simpler relations as shown below,

$$\sum D^i(R)_{mn} \; D^j(R)^*_{mn} = 0 \quad \text{for } i \neq j \qquad \text{(A-2.2)}$$

$$\sum D^i(R)_{mn} \; D^i(R)_{m'n'} = 0 \quad \text{if } m \neq m' \text{ and } n \neq n' \qquad \text{(A-2.3)}$$

$$\sum D^i(R)_{mn} \; D^i(R)^*_{mn} = h^2/l_i \qquad \text{(A-2.4)}$$